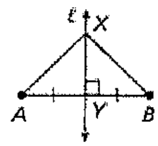
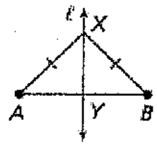
Key

Chapter 5 Notes

5.1 Perpendicular and Angle Bisectors (Book 5.1)

Equidistant: the same distance

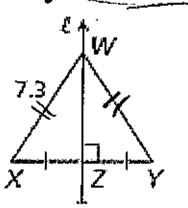
Know it! Note — **Theorems** Distance and Perpendicular Bisectors

	THEOREM	HYPOTHESIS	CONCLUSION
5-1-1	**Perpendicular Bisector Theorem** If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.	 $\overline{XY} \perp \overline{AB}$, $\overline{YA} \cong \overline{YB}$	$XA = XB$
5-1-2	**Converse of the Perpendicular Bisector Theorem** If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.	 $XA = XB$	$\overline{XY} \perp \overline{AB}$, $\overline{YA} \cong \overline{YB}$

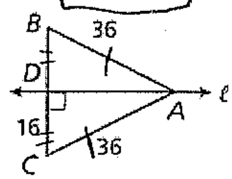
Locus: a set of points that satisfies a condition.

Example #1: Find each measure.

A. $\boxed{YW\ 7.3}$

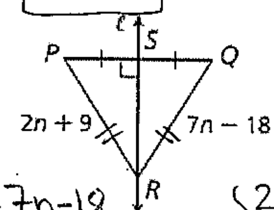


B. $\boxed{BC\ 32}$



16+16=32

C. $\boxed{PR\ 19.8}$



$2n+9=7n-18$

$9=5n-18$

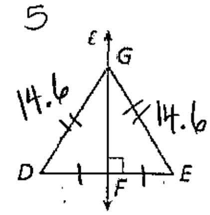
$\frac{27}{5}=\frac{5n}{5}$

$n=5.4$

$2(5.4)+9$

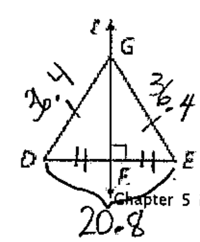
19.8

Example #2: Given that line ℓ is the perpendicular bisector of $\overline{DE}$ and $\overline{EG}$ = 14.6. Find DG. 14.6



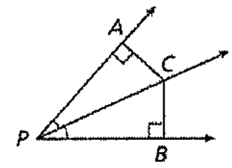
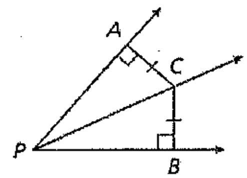
Example #3: Given that DE = 20.8, DG = 36.4, and EG = 36.4, find EF.

$\frac{20.8}{2}=\boxed{10.4}$



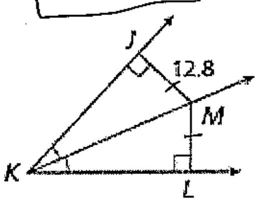
Know it! Note

Theorems: Distance and Angle Bisectors

THEOREM	HYPOTHESIS	CONCLUSION
5-1-3 Angle Bisector Theorem If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.	 $\angle APC \cong \angle BPC$	$AC = BC$
5-1-4 Converse of the Angle Bisector Theorem If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.	 $AC = BC$	$\angle APC \cong \angle BPC$

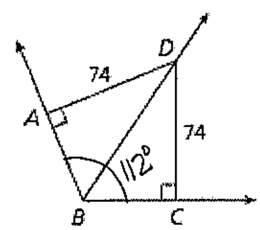
Example #4: Find each measure.

A. LM 12.8

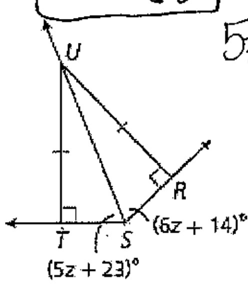


B. $\angle ABD$, given that $\angle ABC = 112°$

$\frac{112}{2} = \boxed{56°}$



C. $\angle TSU$ 68°



$5z+23=6z+14$

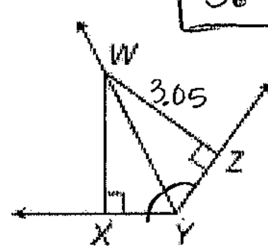
$23=z+14$

$9=z$

$5(9)+23$

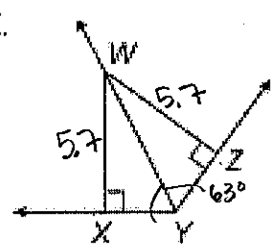
68

D. Given that $\overline{YW}$ bisects $\angle XYZ$ and WZ = 3.05, find WX. $\boxed{3.05}$



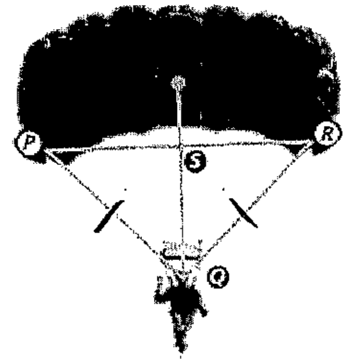
E. Given that $m\angle WYZ = 63°$, XW = 5.7, and ZW = 5.7, find $m\angle XYZ$.

$2(63) = \boxed{126°}$



Example #5:

Each pair of suspension lines on a parachute are the same length and are equally spaced from the center of the chute. How do these lines keep the sky diver centered under the parachute?



Since $\overline{PQ} \cong \overline{RQ}$, we know $\overline{SQ}$ is the $\perp$ bisector.

Therefore, because it's the $\perp$ bisector, we know it's centered.

Example #6:

Write an equation in point-slope form for the perpendicular bisector of the segment with endpoints A(-1,6) and B(3,4).

$y - y_1 = m(x - x_1)$

① find bisector (midpoint):

$\left(\frac{-1+3}{2}, \frac{6+4}{2}\right) = \left(\frac{2}{2}, \frac{10}{2}\right) = \boxed{(1,5)}$

② Find slope:

$\frac{6-4}{-1-3} = \frac{2}{-4} = \frac{-1}{2} \Big\} \perp = 2$

③ equation:

$\boxed{y-5 = 2(x-1)}$

Example #7:

Write an equation in slope-intercept form for the perpendicular bisector of the segment with endpoints P(5,2) and Q(1,-4).

① bisector / midpoint

$\left(\frac{5+1}{2}, \frac{2+-4}{2}\right) = \boxed{(3,-1)}$

② Slope: $\frac{-4-2}{1-5} = \frac{-6}{-4} = \frac{3}{2}$

$\boxed{\perp = \frac{-2}{3}}$

③ equation:

$y+1 = -\frac{2}{3}(x-3)$

$y+1 = -\frac{2}{3}x + 2$

$\boxed{y = -\frac{2}{3}x - 1}$

5.2 Bisectors of Triangles (Book 5.2)

Circumcenter Construction Project

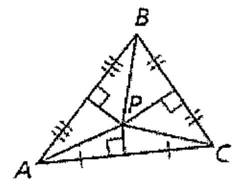
Where three or more lines intersect at one point, the lines are said to be Concurrent. The point of concurrency is the point where they intersect. The point of concurrency is the circumcenter of the Δ.

Know it! Note

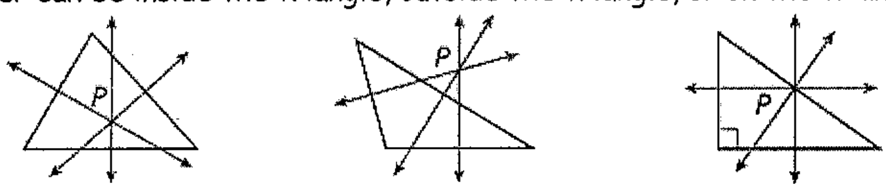
Theorem 5-2-1 Circumcenter Theorem

The circumcenter of a triangle is equidistant from the vertices of the triangle.

$$PA = PB = PC$$



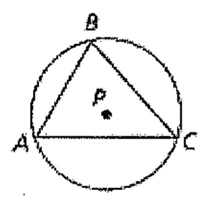
The circumcenter can be inside the triangle, outside the triangle, or on the triangle.



Acute triangle — Obtuse triangle — Right triangle

The circumcenter of ΔABC is the center of its circumscribed circle.

A circle that contains all the vertices of a polygon is circumscribed about the polygon.

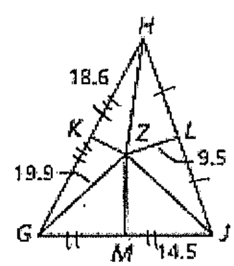


Example #1:

$\overline{KZ}, \overline{LZ}, \overline{MZ}$ are the perpendicular bisectors of ΔGHJ. Find HZ.

→ pt Z is equidistant from each vertice: $\overline{HZ} \cong \overline{JZ} \cong \overline{GZ}$

$\boxed{HZ = 19.9}$



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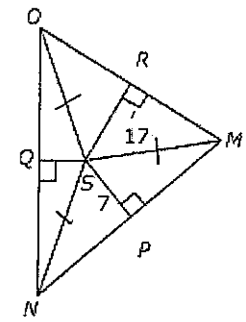
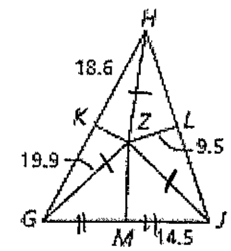
Example #2: $\overline{RS}, \overline{QS}, \overline{PS}$ are the perpendicular bisectors of ΔMNO. Find NS.



$\boxed{NS = 17}$

Example #3: $\overline{KZ}, \overline{LZ}, \overline{MZ}$ are the perpendicular bisectors of ΔGHJ. Find:

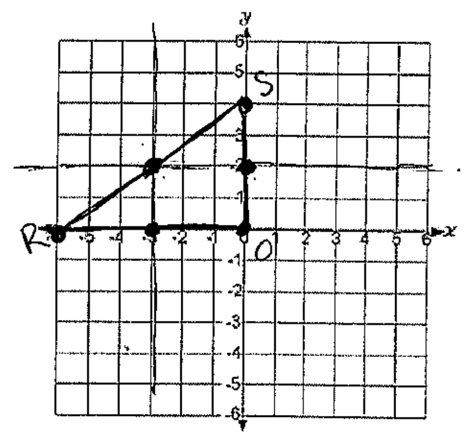


A. GM 14.5

B. GK 18.6

C. JZ 19.9

Example #4: Find the circumcenter of ΔRSO with vertices R(-6,0), S(0,4), and O(0,0).

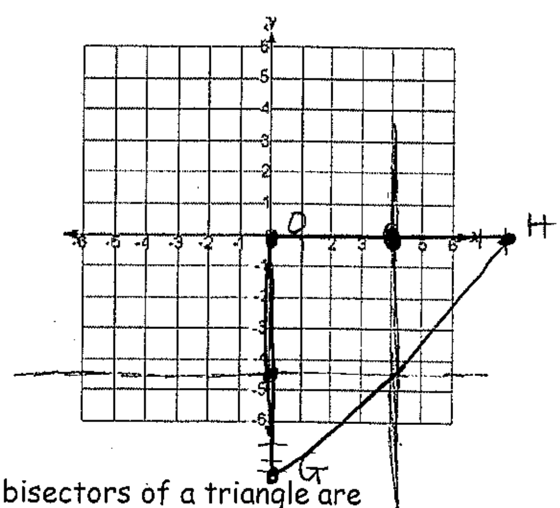


① Find 2 equations for ⊥ bisectors: $x = -3$ & $y = 2$

② Find the intersection of the equations

$\boxed{(-3, 2)}$

Example #5: Find the circumcenter of ΔGOH with vertices G(0,-9), O(0,0), and H(8,0).



⊥ bisectors equations: $X = 4$ & $y = -4.5$

Circumcenter: $\boxed{(4, -4.5)}$

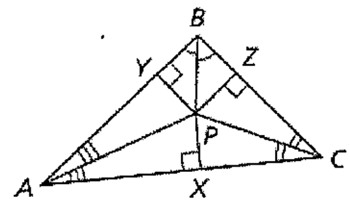
A triangle has three angles, so it has angle bisectors. The angle bisectors of a triangle are also concurrent. This point of concurrency is the Incenter of the Δ.

Know it! Note

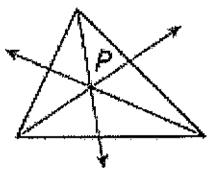
Theorem 5-2-2 **Incenter Theorem**

The incenter of a triangle is equidistant from the sides of the triangle.

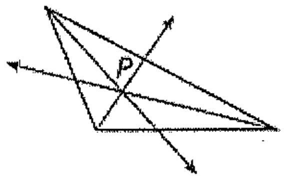
$PX = PY = PZ$



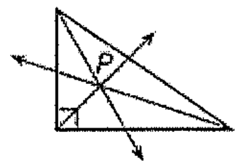
Unlike the circumcenter, the incenter is always inside the triangle.



Acute triangle



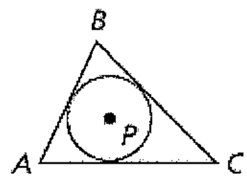
Obtuse triangle



Right triangle

The incenter is the center of the triangle's inscribed circle.

A circle inscribed in a polygon intersects each line that contains a side of the polygon at exactly one point.



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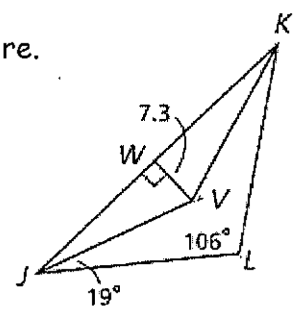
Example #6: $\overline{JV}$ and $\overline{KV}$ are angle bisectors of ΔJKL. Find each measure.

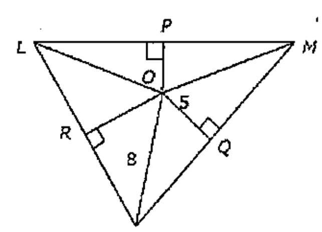
A. the distance from V to $\overline{KL}$ 7.3

B. m∠VKL $\frac{180-[2\cdot 19]-106}{2}=\frac{36}{2}=\boxed{18^\circ}$



Example #7: $\overline{KO}, \overline{MO}, \overline{LO}$ are the angle bisectors of ΔKLM. Identify the distance from O to $\overline{LM}$.

$\boxed{5}$



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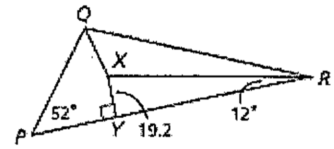
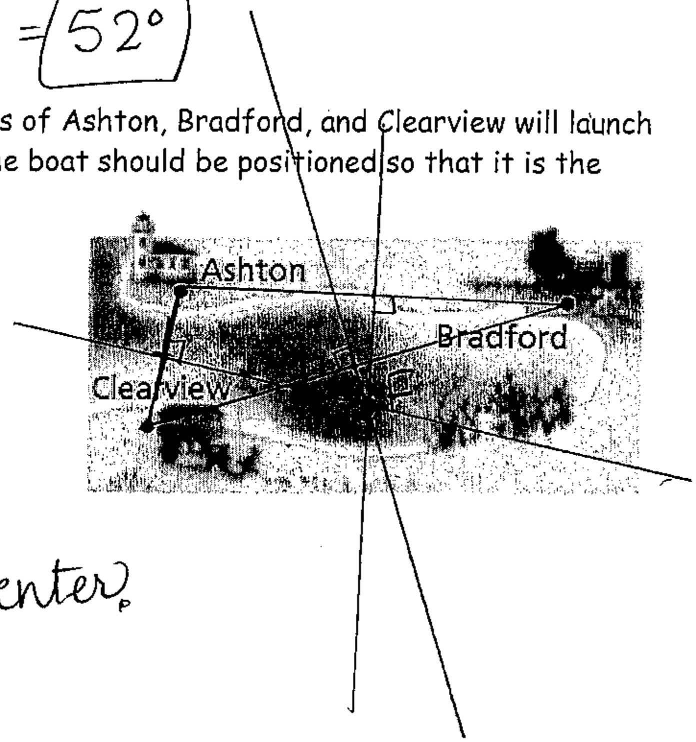
Example #8: $\overline{QX}$ and $\overline{RX}$ are angle bisectors of $\triangle PQR$. Find each measure.



A. the distance from X to $\overline{PQ}$ $\boxed{19.2}$

B. m∠PQX $\frac{180-52-[2\cdot 12]}{2} = \frac{104^\circ}{2} = \boxed{52^\circ}$

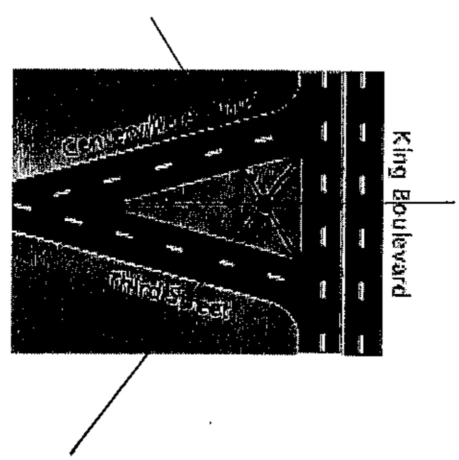
Example #9: For the next Fourth of July, the towns of Ashton, Bradford, and Clearview will launch a fireworks display. Draw a sketch to show where the boat should be positioned so that it is the same distance from all three towns.



* Find the ⊥ bisector of each side
* Where the lines intersect is the circumcenter.

- F is the circumcenter.

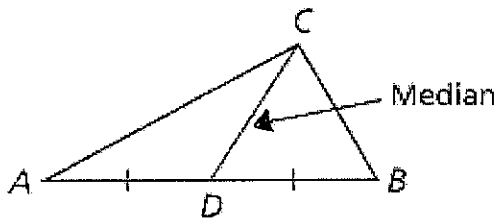
Example #10: A city plans to build a firefighters' monument in the park between three streets. Draw a sketch to show where the city should place the monument so that it is the same distance from all three streets.



* Draw angle bisectors to find the incenter.

5.3 Medians and Altitudes of Triangles (Book 5.3)

A median of a Δ is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side.



* Every triangle has three medians, and the medians are concurrent.

* The point of concurrency of the medians of a triangle is the Centroid of the Δ.

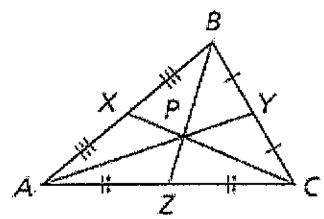
* The centroid is always inside the triangle. The centroid is also called the center of gravity because it is the point where a triangular region will balance.

Know it! Note

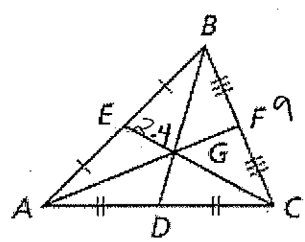
Theorem 5-3-1 Centroid Theorem

The centroid of a triangle is located $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.

$AP = \frac{2}{3}AY$ $\quad$ $BP = \frac{2}{3}BZ$ $\quad$ $CP = \frac{2}{3}CX$



Example #1: ΔABC, AF = 9, and GE = 2.4. Find each length.



A. AG $\left(\frac{2}{3}\right)(9) = \boxed{6}$

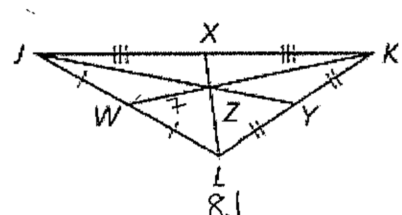
B. CE
$CG + GE = CE$
$\frac{2}{3}CE + 2.4 = CE$
$2.4 = \frac{1}{3}CE$ $\quad \boxed{CE = 7.2}$

OR $(2.4)(3) = \boxed{7.2}$

Example #2: In ΔJKL, ZW = 7, and LX = 8.1. Find each length.

A. KW $(7)(3) = \boxed{21}$

B. LZ $\left(\frac{2}{3}\right)(8.1) = \boxed{5.4}$



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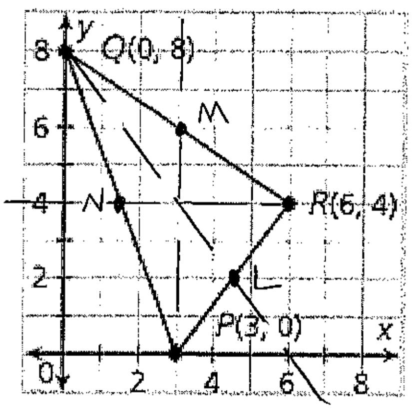
Example #3:



The diagram shows the plan for a triangular piece of a mobile. Where should the sculptor attach the support so that the triangle is balanced. *Find the coordinates of the centroid.*

① Find midpts:

$\overline{QR}$: $\left(\frac{0+6}{2}, \frac{8+4}{2}\right) = (3,6)$ M

$\overline{RP}$: $\left(\frac{6+3}{2}, \frac{4+0}{2}\right) = (4.5, 2)$ L

$\overline{QP}$: $\left(\frac{0+3}{2}, \frac{8+0}{2}\right) = (1.5, 4)$ N

② Equations:

$\overline{PM}$ = $x = 3$

$\overline{RN}$ = $y = 4$

$\boxed{\text{Centroid: } (3,4)}$

An altitude is a perpendicular segment from a vertex to the line containing the opposite side. Every triangle has three altitudes. An altitude can be inside, outside, or on the triangle.

Hint: * the height of a Δ is the length of an altitude.

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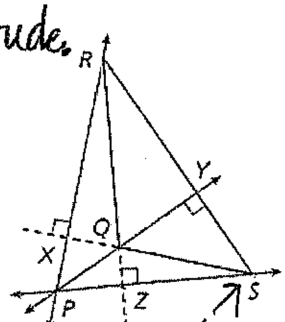
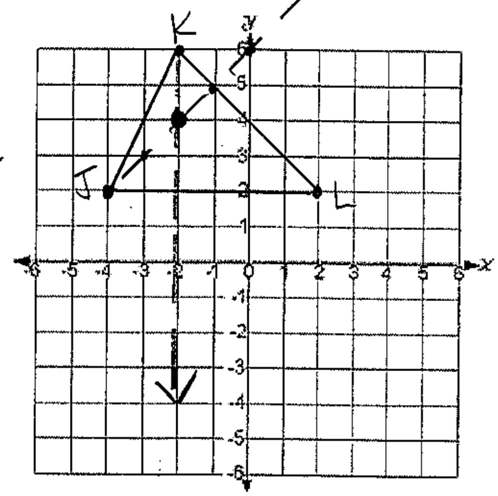
In ΔQRS, altitude $\overline{QY}$ is inside the triangle, but $\overline{RX}$ and $\overline{SZ}$ are not.



Notice that all the lines containing the altitudes are concurrent at P.

This point of concurrency is the orthocenter of the Δ

Example #4: Find the orthocenter of ΔJKL with vertices J(-4,2), K(-2,6), and L(2,2).



① Find an equation containing the altitude from K to $\overline{JL}$

$x = -2$

② Find equation from J to $\overline{KL}$: Slope of $\overline{KL}$ $\frac{6-2}{-2-2} = \frac{4}{-4} = -1$; ⊥ slope = 1

use J(-4,2) $y-2 = 1(x+4)$

$y-2 = x+4$

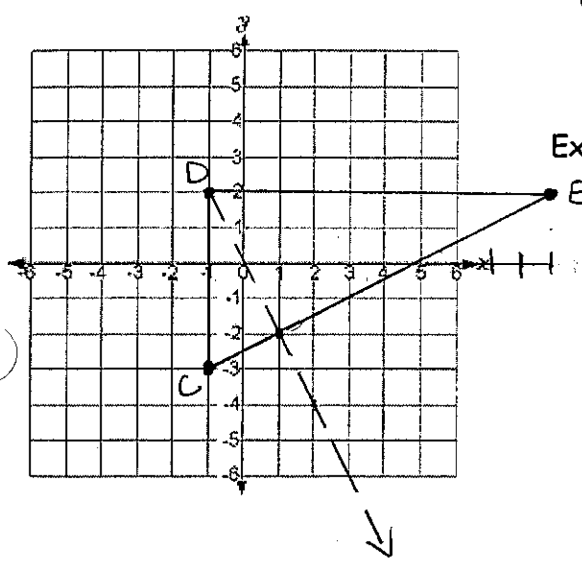
$y = x+6$

③ Solve $\begin{cases} x = -2 \\ y = x+6 \end{cases}$

$y = -2+6$

$y = 4$

$\boxed{(-2,4)}$



Example #5: Find the orthocenter of ΔCDE with vertices C(-1,-3), D(-1,2) and E(9,2).

$y = 2$

D to $\overline{CE}$: Slope of $\overline{CE}$: $\frac{-3-2}{-1-9} = \frac{-5}{-10} = \frac{1}{2}$; ⊥ slope = -2

D(-1,2) $y-2 = -2(x+1)$

$y-2 = -2x-2$

$y = -2x$

$\begin{cases} y = 2 \\ y = -2x \end{cases}$

$\frac{2}{-2} = \frac{-2x}{-2}$ $x = -1$

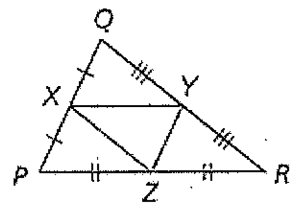
$\boxed{(-1,2)}$

Chapter 5 Notes

5.4 Triangle Mid Segment Theorem (Book 5.4)

Mid segment of a triangle :

a segment that joins the midpts of 2 sides of a $\triangle$



Midsegments: $\overline{XY}$, $\overline{YZ}$, $\overline{ZX}$

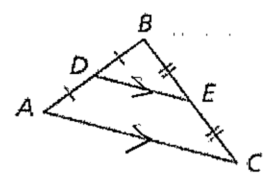
Midsegment triangle: $\triangle XYZ$

Know it! Note

Theorem 5-4-1 **Triangle Midsegment Theorem**

A midsegment of a triangle is parallel to a side of the triangle, and its length is half the length of that side.

$\overline{DE} \parallel \overline{AC}$, $DE = \frac{1}{2}AC$

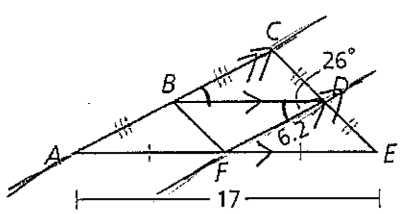


b/c lines are // there will be angle measures we can find

Example #1:

A) BD $\frac{17}{2} = \boxed{8.5}$

B) $m\angle CBD$ $\boxed{26^\circ}$

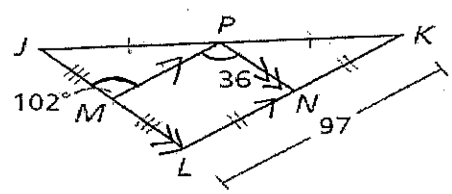


C) JL $2 \cdot 36 = \boxed{72}$

D) PM $\frac{97}{2} = \boxed{48.5}$

E) $m\angle MPN$ $\boxed{102^\circ}$

F) $m\angle PNL$ $180-102$ $\boxed{78^\circ}$



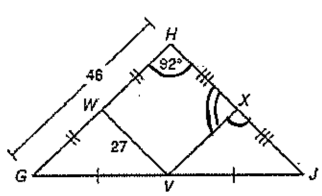
G) VX $\frac{46}{2} = \boxed{23}$

H) HJ $27 \cdot 2 = \boxed{54}$

K) $m\angle VXJ$ $\boxed{92^\circ}$

L) XJ $\boxed{27}$

O) $m\angle HXV$ $180-92 = \boxed{88^\circ}$



I) ST $36 \cdot 2 = \boxed{72}$

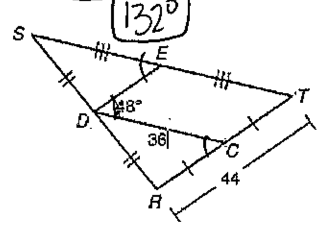
J) DE $\frac{44}{2} = \boxed{22}$

M) $m\angle DES$ $\boxed{48^\circ}$

N) $m\angle RCD$ $\boxed{48^\circ}$

P) $m\angle DCT$ $180 - 48$ $\boxed{132^\circ}$

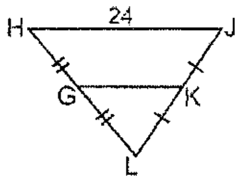
Q) $m\angle STR$ $180 - 132$ $\boxed{48^\circ}$



Chapter 5 Notes

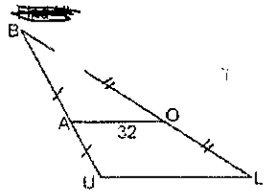
Example #2: Find each measure

A) Find GK. $\boxed{12}$



$\frac{24}{2}$

B) Find UL. $32 \cdot 2 = \boxed{64}$

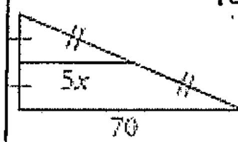


C) Find x.

$2(5x) = 70$

$10x = 70$

$\boxed{x = 7}$

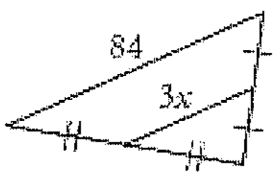


D) Find x.

$2(3x) = 84$

$6x = 84$

$\boxed{x = 14}$



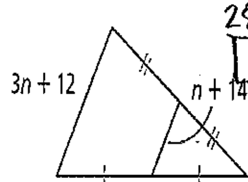
E) Find n.

$2(n+14) = 3n+12$

$2n + 28 = 3n + 12$

$28 = n + 12$

$\boxed{16 = n}$



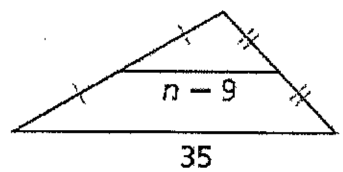
F) Find n.

$2(n-9) = 35$

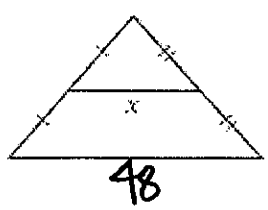
$2n - 18 = 35$

$2n = 53$

$\boxed{n = 26.5}$



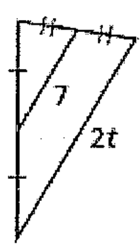
G) Find x.



$2x = 48$

$\boxed{x = 24}$

H) Find t.



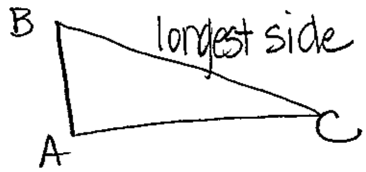
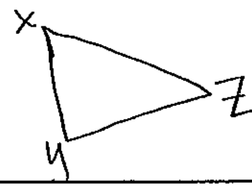
$2(7) = 2t$

$14 = 2t$

$\boxed{7 = t}$

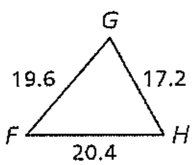
5.5 Inequalities in Triangles (Book 5.5)

Theorems (Angle Side Relationships in Triangles)

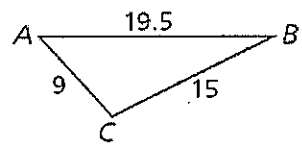
Theorem	Hypothesis	Conclusion
If two sides of a triangle are not congruent, then the larger angle is opposite the longer side. (In △, larger ∠ is opp. longer side.)		∠A is the largest angle
If two angles of a triangle are not congruent, then the longer side is opposite the larger angle. (In △, longer side is opp. larger ∠.)	 y is the largest ∠.	$\overline{XZ}$ is the longest side.

Example #1: Write the angles in order from smallest to largest.

A) ∠F, ∠H, ∠G

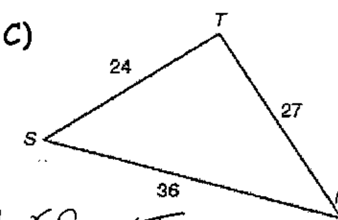


B)



∠B, ∠A, ∠C

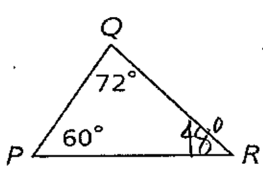
C)



∠V, ∠S, ∠T

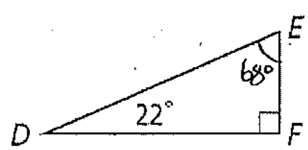
Example #2: Write the side lengths in order from smallest to largest.

A)



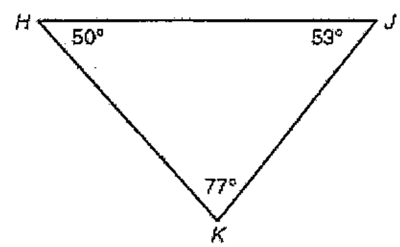
$\overline{PQ}$, $\overline{QR}$, $\overline{PR}$

B)



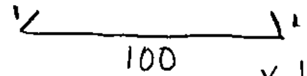
$\overline{EF}$, $\overline{DF}$, $\overline{DE}$

C)



$\overline{KJ}$, $\overline{HK}$, $\overline{HJ}$

Sketch a Δ with sides $100, 1, 1$



* left & right sum > 100

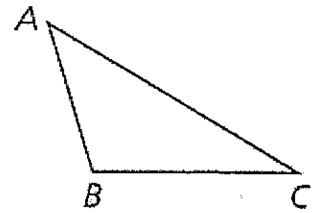
Chapter 5 Notes

Theorem 5-5-3 Triangle Inequality Theorem

Theorem 5-5-3 **Triangle Inequality Theorem**

The sum of any two side lengths of a triangle is greater than the third side length.

$$AB + BC > AC$$
$$BC + AC > AB$$
$$AC + AB > BC$$



Example #3: Applying the Triangle Inequality Theorem - Tell whether a triangle can have sides with the given lengths.

A) 7, 10, 19
$17 < 19$
not a Δ

B) 2.3, 3.1, 4.6
$5.4 > 4.6$
yes Δ

C) 8, 13, 21
$21 = 21$
not a Δ

D) 6.2, 7, 9
$13.2 > 9$
yes Δ

E) 3, 5, 8
$8 = 8$
not a Δ

F) 11, 15, 21
$26 > 21$
yes Δ

Example #4: Finding Side Lengths - The lengths of two sides of a triangle are given.
Find the range of possible lengths for the third side.

* Subtract to get lower boundary
* add to get upper boundary

A) 8 in and 13 in
$5 < x < 21$

B) 22 in and 17 in
$5 < x < 39$

C) 4 in and 10 in
$6 < x < 14$

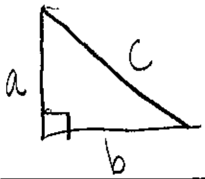
D) 8 ft and 8 ft
$0 < x < 16$

E) 6.2 cm and 12 cm
$5.8 < x < 18.2$

F) 3.2 in and 1.4 in
$1.8 < x < 4.6$

5.6 The Pythagorean Theorem (Book 5.7)

Pythagorean Theorem



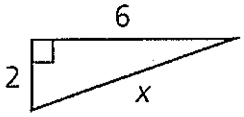
a & b = legs

c = hypotenuse
= longest side

$a^2+b^2=c^2$

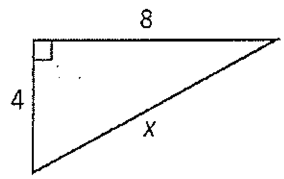
Example #1: Use the Pythagorean Theorem to find x. Write your answer in simplified radical form.

A)



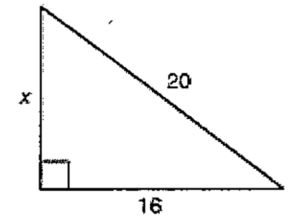
$2^2+6^2=x^2$
$4+36=x^2$
$\sqrt{40}=\sqrt{x^2}$
$\boxed{x=2\sqrt{10}}$

B)



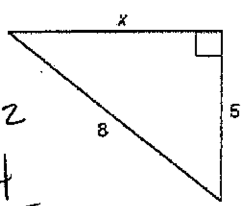
$4^2+8^2=x^2$
$16+64=x^2$
$\sqrt{80}=\sqrt{x^2}$
$x=\sqrt{16\cdot 5}$
$\boxed{x=4\sqrt{5}}$

C)



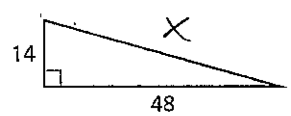
$x^2+16^2=20^2$
$x^2+256=400$
$\sqrt{x^2}=\sqrt{144}$
$\boxed{x=12}$

D)



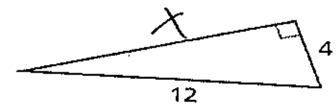
$x^2+5^2=8^2$
$x^2+25=64$
$\sqrt{x^2}=\sqrt{39}$
$\boxed{x=\sqrt{39}}$

E)



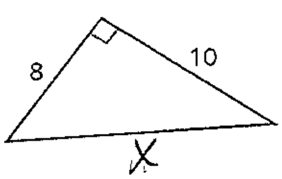
$14^2+48^2=x^2$
$196+2304=x^2$
$\sqrt{2500}=\sqrt{x^2}$
$\boxed{50=x}$

F)



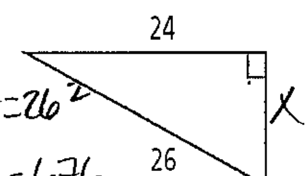
$x^2+4^2=12^2$
$x^2+16=144$
$x^2=128$
$\sqrt{x^2}=\sqrt{64\cdot 2}$
$\boxed{x=8\sqrt{2}}$

G)



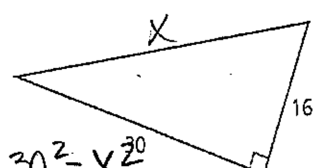
$8^2+10^2=c^2$
$64+100=c^2$
$164=c^2$
$\sqrt{4\cdot 41}=\sqrt{c^2}$
$\boxed{2\sqrt{41}=c}$

H)



$x^2+24^2=26^2$
$x^2+576=676$
$\sqrt{x^2}=\sqrt{100}$
$x=100$

I)

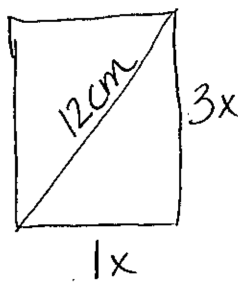


$16^2+30^2=x^2$
$256+900=x^2$
$\sqrt{1156}=\sqrt{x^2}$
$\boxed{34=x}$

Example #2:

length width

A) Randy is building a rectangular picture frame. He wants the ratio of the length to the width to be 3:1 and the diagonal to be 12 centimeters. How wide should the frame be? Round to the nearest tenth of a centimeter.



$(3x)^2 + x^2 = 12^2$

$9x^2 + x^2 = 144$

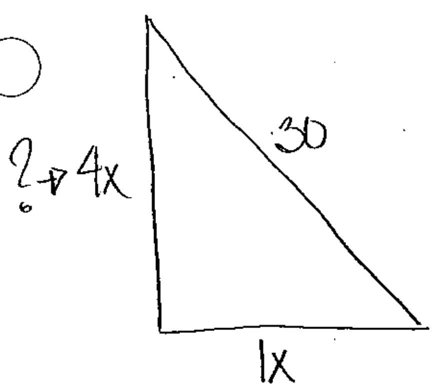
$\frac{10x^2}{10} = \frac{144}{10}$

$\sqrt{x^2} = \sqrt{14.4}$

$\boxed{x = 3.8\text{cm}} \rightarrow$ width

length $= 3(3.8)$
$= 11.4$cm

B) According to the recommended safety ratio of 4:1, how high will a 30 foot ladder reach when placed against a wall? Round to the nearest inch?



$x^2 + (4x)^2 = 30^2$

$x^2 + 16x^2 = 900$

$17x^2 = 900$

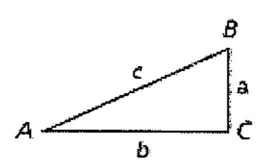
$\sqrt{x^2} = \sqrt{\frac{900}{17}}$

$x = 7.3$

$4(7.3) = \boxed{29.2\text{ ft}}$

Know it! Note

Theorems 5-7-1 Converse of the Pythagorean Theorem

THEOREM	HYPOTHESIS	CONCLUSION
If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.	 $a^2 + b^2 = c^2$	$\triangle ABC$ is a right triangle.

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P.T. Inequality Theorem

$c^2 > a^2 + b^2$

obtuse $\triangle$

$c^2 < a^2 + b^2$

acute $\triangle$

Chapter 5 Notes

5.7 45/45/90 Triangles (Book 5.8)

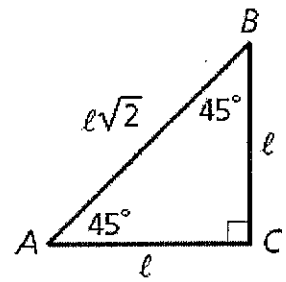
look @ book 5-8

Theorem 5-8-1 **45°-45°-90° Triangle Theorem**

In a 45°-45°-90° triangle, both legs are congruent, and the length of the hypotenuse is the length of a leg times $\sqrt{2}$.

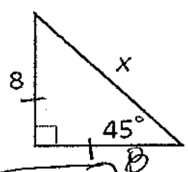
$AC = BC = \ell$ $\qquad$ $AB = \ell\sqrt{2}$

legs are $\cong$ $\qquad$ hypotenuse leg$\sqrt{2}$



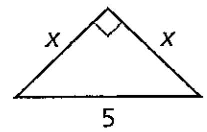
Example #1: Use special right triangles to find x.

A)



$\boxed{8\sqrt{2}}$

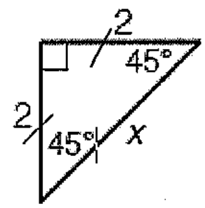
B)



$\frac{5}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$

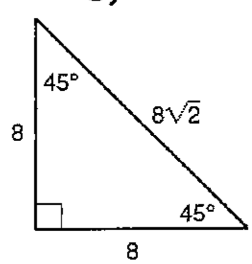
$\frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{5\sqrt{2}}{2}}$

C)



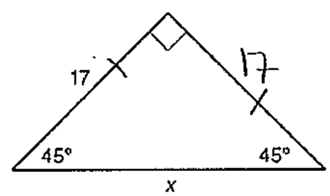
$\boxed{2\sqrt{2}}$

D)



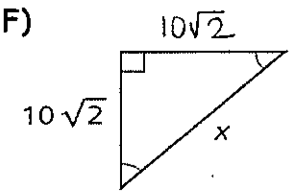
already done

E)



$\boxed{17\sqrt{2}}$

F)



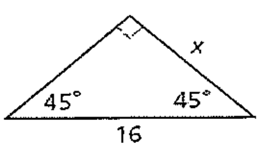
$10\sqrt{2} \cdot \sqrt{2}$

$10\sqrt{4}$

$10 \cdot 2$

$\boxed{20}$

G)

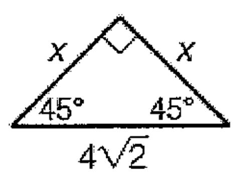


$\frac{16}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$

$\frac{16}{\sqrt{2}} = x$

$\frac{16}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{16\sqrt{2}}{2} = \boxed{8\sqrt{2}}$

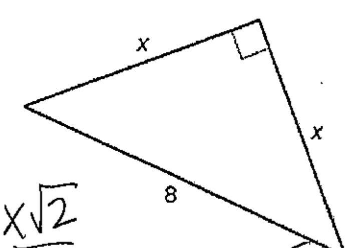
H)



$\frac{4\sqrt{2}}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$

$\boxed{4 = x}$

I)



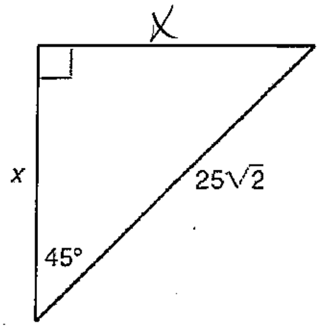
$\frac{8}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$

$\frac{8}{\sqrt{2}} = x$

$\frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2}$

$\boxed{4\sqrt{2}}$

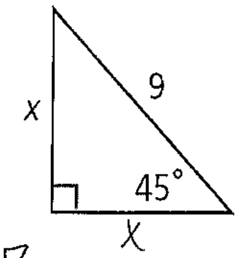
J)



$$\frac{25\sqrt{2}}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$$

$$\boxed{25 = x}$$

K)

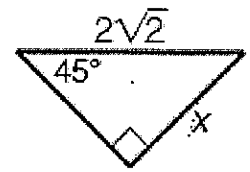


$$\frac{9}{\sqrt{2}} = \frac{x\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$$

$$\frac{9}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\boxed{\frac{9\sqrt{2}}{2}} = x$$

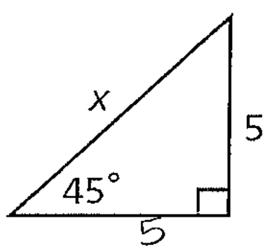
L)



$$\frac{2\sqrt{2}}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$$

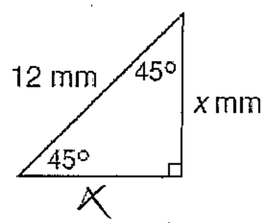
$$\boxed{2 = x}$$

M)



$$\boxed{5\sqrt{2}}$$

N)

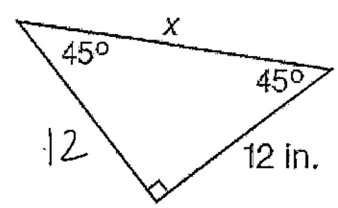


$$\frac{12}{\sqrt{2}} = \frac{x\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$$

$$\frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{2}$$

$$\boxed{6\sqrt{2}}$$

O)



$$\boxed{x = 12\sqrt{2}}$$

5.8 30/60-90 Triangles (Book 5.8)

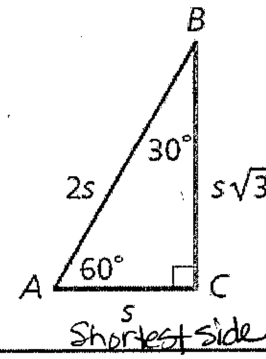
Theorem 5-8-2 **30°-60°-90° Triangle Theorem**

In a 30°-60°-90° triangle, the length of the hypotenuse is is 2 times the length of the shorter leg, and the length of the longer leg is the length of the shorter leg times $\sqrt{3}$.

$AC = s$ Shortest leg

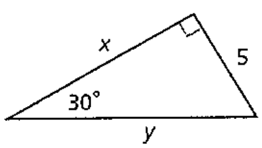
$AB = 2s$ hypotenuse

$BC = s\sqrt{3}$ Other leg



Example #1: Use special right triangles to find x and y.

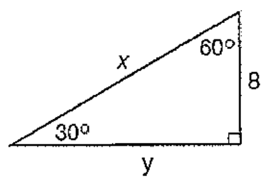
A)



$y = 5 \cdot 2$
$\boxed{= 10}$

$\boxed{x = 5\sqrt{3}}$

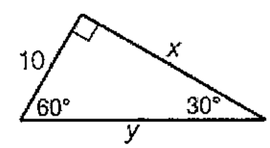
B)



$x = 8 \cdot 2$
$\boxed{= 16}$

$y = 8\sqrt{3}$

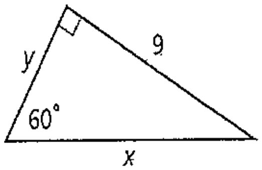
C)



$y = 10 \cdot 2$
$\boxed{= 20}$

$x = \boxed{10\sqrt{3}}$

D)



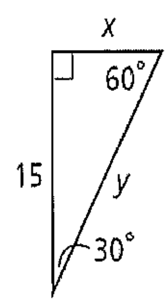
$\frac{9}{\sqrt{3}} = \frac{y\sqrt{3}}{\sqrt{3}}$

$\frac{9\sqrt{3}}{3} = y$

$\boxed{3\sqrt{3} = y}$

$x = 2 \cdot 3\sqrt{3}$
$= \boxed{6\sqrt{3}}$

E)



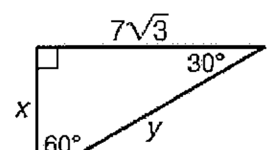
$\frac{15}{\sqrt{3}} = \frac{x\sqrt{3}}{\sqrt{3}}$

$\frac{15\sqrt{3}}{3} = x$

$\boxed{5\sqrt{3}}$

$y = 2 \cdot 5\sqrt{3}$
$= \boxed{10\sqrt{3}}$

F)

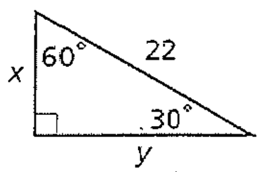


$x = \frac{7\sqrt{3}}{\sqrt{3}}$

$x = 7$

$y = 7 \cdot 2$
$= 14$

G)

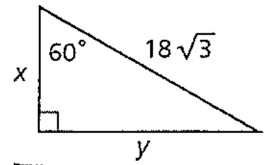


$x = \frac{22}{2}$

$\boxed{x = 11}$

$\boxed{y = 11\sqrt{3}}$

H)



$x = \frac{18\sqrt{3}}{2}$

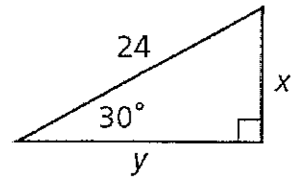
$\boxed{x = 9\sqrt{3}}$

$y = 9\sqrt{3} \cdot \sqrt{3}$

$= 9\sqrt{9}$

$\boxed{= 27}$

I)

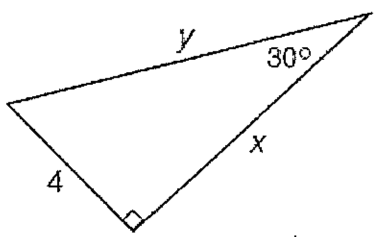


$x = \frac{24}{2}$

$\boxed{= 12}$

$\boxed{y = 12\sqrt{3}}$

J)

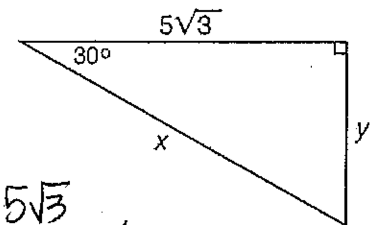


$y = 4 \cdot 2$

$\boxed{= 8}$

$\boxed{x = 4\sqrt{3}}$

K)



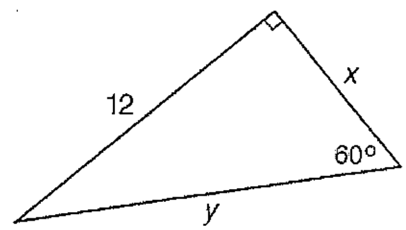
$y = \frac{5\sqrt{3}}{\sqrt{3}}$

$\boxed{y = 5}$

$x = 5 \cdot 2$

$\boxed{= 10}$

L)



$x = \frac{12}{\sqrt{3}}$

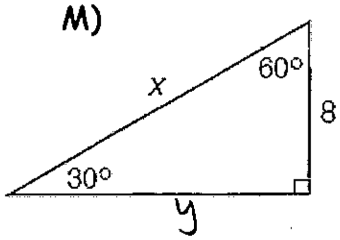
$= \frac{12\sqrt{3}}{3}$

$= \boxed{4\sqrt{3}}$

$y = 2 \cdot 4\sqrt{3}$

$\boxed{= 8\sqrt{3}}$

M)



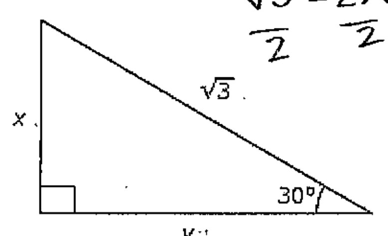
$x = 8 \cdot 2$

$\boxed{= 16}$

$\boxed{y = 8\sqrt{3}}$

N)

$\frac{\sqrt{3}}{2} = \frac{2x}{2}$

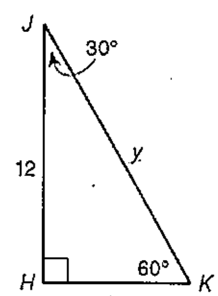


$x = \boxed{\frac{\sqrt{3}}{2}}$

$y = \sqrt{3} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{9}}{2}$

$\boxed{y = \frac{3}{2}}$

O)



$x = \frac{12}{\sqrt{3}}$

$= \frac{12\sqrt{3}}{3}$

$\boxed{= 4\sqrt{3}}$

$y = 2 \cdot 4\sqrt{3}$

$\boxed{= 8\sqrt{3}}$