Classroom Copy

Chapter 4 Notes

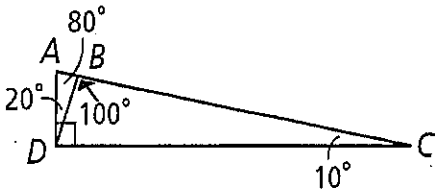
4-1 Classifying Triangles (book 4.1)

Classifying Triangles by Angle Measure

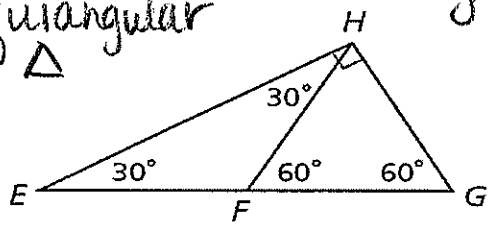
Acute Triangle	Equiangular Triangle	Right Triangle	Obtuse Triangle
			
3 acute ∠'s	3 ≅ ∠'s	1 Right ∠	1 obtuse ∠

Example #1 Classify each triangle by its angle measures.

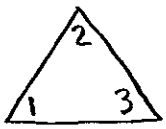
A. △BDC. obtuse △ B. △ABD acute △



C. △FHG equiangular △ D. △EHG Right △



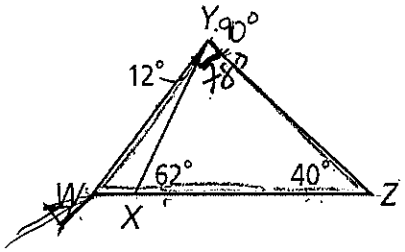
Theorem 4-2-1 Triangle Sum Theorem



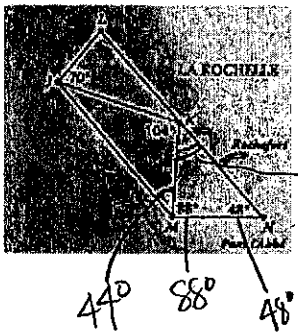
$\angle 1 + \angle 2 + \angle 3 = 180°$

Example #2 Find the following angle measures.

A. ∠XYZ 78° B. ∠YWZ 50°



C. ∠MKN 44° D. ∠MJK 32°

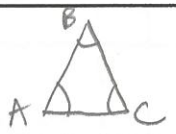


180
−104
− 44

Chapter 4 Notes

Vocabulary: Corollary mini theorem

Corollaries

Corollary	Hypothesis	Conclusion
Acute angles of a Right △ are Complementary.		$\angle 1+\angle 2=90°$
		all angles = 60° $\angle A=60°$ $\angle B=60°$ $\angle C=60°$

Example #3 The measure of one of the acute angles in a right triangle is given. What is the measure of the other acute angle?

A. 63.7° B. 2x° C. 48 2/5° D. x°

$63.7+X=90$

$26.3°$

$(90-2x)°$

48.4

$41.6°$

$41\frac{3}{5}°$

$(90-x)°$

Vocabulary

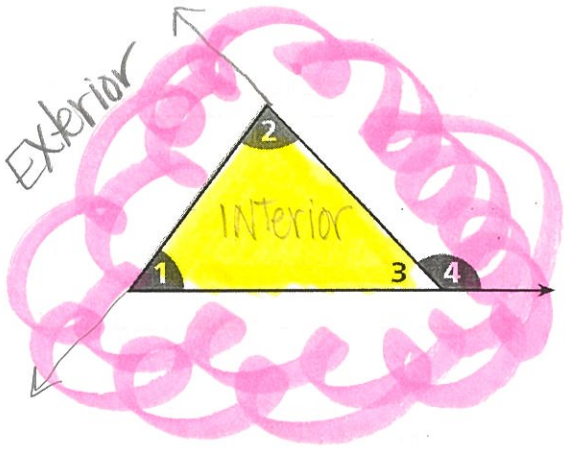
Interior

Exterior

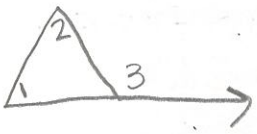
Interior Angle $\angle 1, \angle 2, \angle 3$

Exterior Angle $\angle 4$

"far away" Remote Interior Angle $\angle 1$ & $\angle 2$



Theorem 4-2-4 Exterior Angle



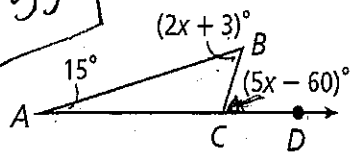
The exterior angle is the sum of the remote interior angles.

$\angle 3=\angle 1+\angle 2$

Chapter 4 Notes

Example #4 Find each angle.

A. Find $m\angle B$ — 55°



$5x-60=(2x+3)+15$

$5x-60=2x+18$

$3x-60=18$

$\frac{3x}{3}=\frac{78}{3}$

$x=26$

$2(26)+3=55$

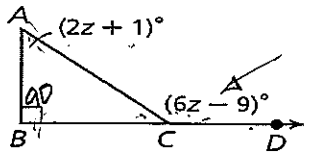
B. Find $m\angle ACD$ — 141°

$6z-9=2z+91$

$4z-9=91$

$4z=100$

$z=25$



$6(25)-9$

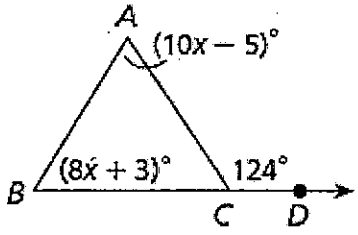
C. Find $m\angle B$ — 59°

$124=18x-2$

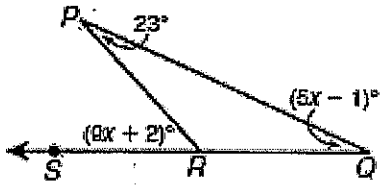
$126=18x$

$7=x$

$8(7)+3=$



D. Find $m\angle PRS$ — 47°



$9x+2=5x+22$

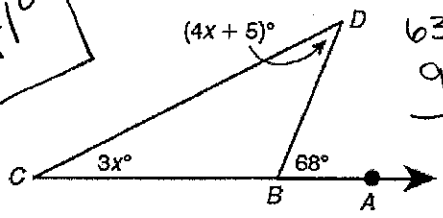
$4x+2=+22$

$4x=20$

$x=5$

$9(5)+2$

E. Find $m\angle D$ — 41°



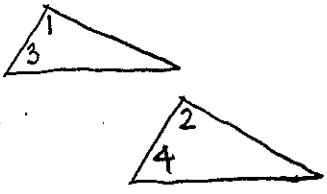
$68=7x+5$

$63=7x$

$9=x$

$4(9)+5$

Theorem 4-2-5 Third Angles Theorem

Theorem	Hypothesis	Conclusion
		If 2 ∠'s of one △ are ≅ to 2 ∠'s of another △, the third ∠'s are ≅.

$\angle 1 \cong \angle 2$

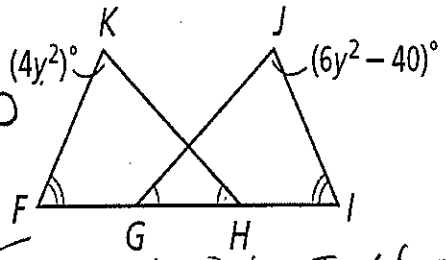
$\angle 3 \cong \angle 4$

Chapter 4 Notes

Example #5 Find each angle.

A. Find $m\angle K$ and $m\angle J$

80° 80°



$4y^2 = 6y^2 - 40$

$-2y^2 = -40$

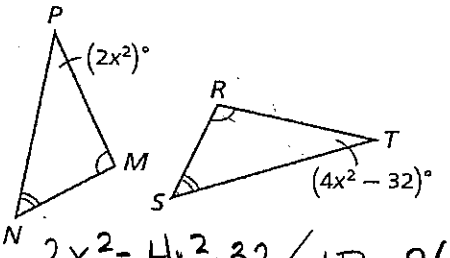
$y^2 = 20$

$\angle K = 4(20)$
$= 80$

$\angle J = 6(20) - 40$
$= 80$

B. Find $m\angle P$ and $m\angle T$

32° 32°



$2x^2 = 4x^2 - 32$

$-2x^2 = -32$

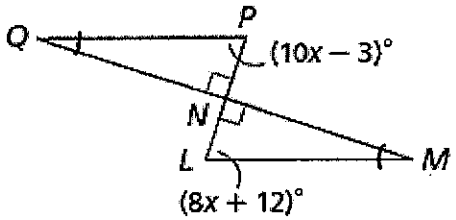
$x^2 = 16$

$\angle P = 2(16)$
$= 32$

$\angle T = (4(16) - 32)$
$= 32$

C. Find $m\angle P$ and $m\angle L$

72° 72°



$10x - 3 = 8x + 12$

$2x - 3 = 12$

$2x = 15$

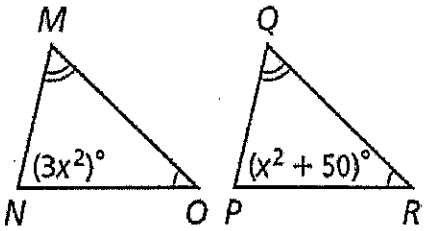
$x = 7.5$

$\angle P = 10(7.5) - 3$
$= 72$

$\angle L = 8(7.5) + 12$
$= 72$

D. Find $m\angle P$ and $m\angle N$

75° 75°



$3x^2 = x^2 + 50$

$2x^2 = 50$

$x^2 = 25$

$\angle P = 25 + 50$
$= 75$

$\angle N = 3(25)$
$= 75$

Chapter 4 Notes

4-2 Angle Relationships in Triangles (book 4.2)

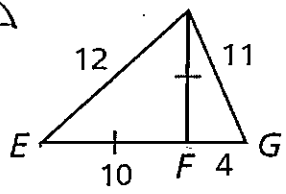
Classifying Triangles by Side Length

Equilateral Triangle	Isosceles Triangle	Scalene Triangle
		
3 ≅ sides	2 ≅ sides	no ≅ sides

Example #1 Classify each triangle by its side lengths.

A. △EHF

Isosceles △

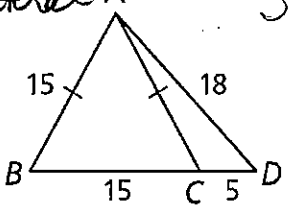


B. △EHG

Scalene △

C. △ABC

Equilateral △

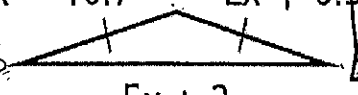


D. △ACD

Scalene △

Example #2 Find the side lengths of each of the following triangles.

A.



$4x - 10.7 = 2x + 6.3$

$2x - 10.7 = 6.3$

$2x = 17$

$x = 8.5$

JK = 23.3

KL = 23.3

JL = 44.5

B. △FGH is equilateral

FG = 17

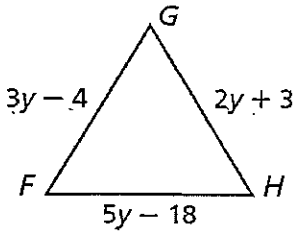
GH = 17

FH = 17

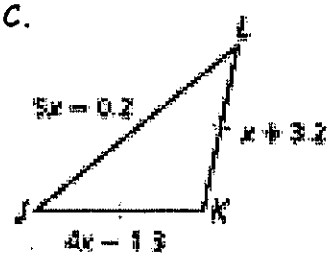
$3y - 4 = 2y + 3$

$y - 4 = 3$

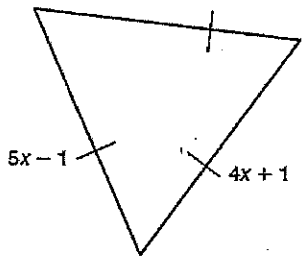
$y = 7$



C.



D.



Chapter 4 Notes

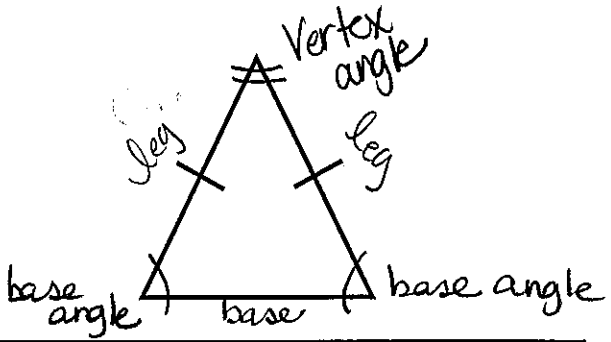
Vocabulary - label the following triangle

leg

Vertex angle

base

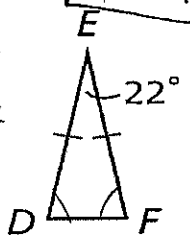
base angle



Isosceles Triangle Theorem If 2 sides of a Δ are ≅, then the opposite ∠'s are ≅.	if △	then △
Converse of the Isosceles Triangle Theorem If 2 ∠'s of a Δ are ≅, then the 2 opposite sides are ≅.	if △	then △

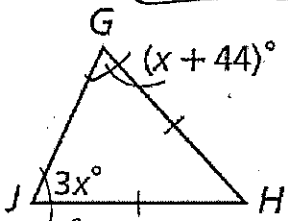
Example #3 Find each angle measure.

A. m∠F 79°



$\frac{180-22}{2} =$

B. m∠G 66°

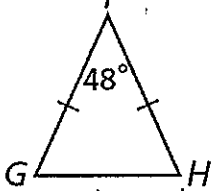


$3x = x + 44$

$2x = 44$

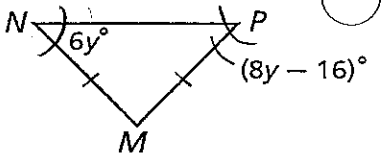
$x = 22$

C. m∠H 66°



$\frac{180-48}{2} = 66$

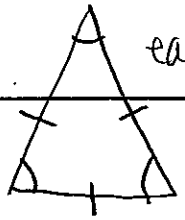
D. m∠N 48°



$6y = 8y - 16$

$-2y = -16$

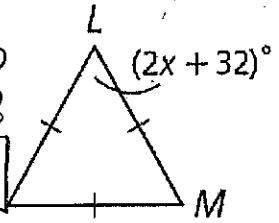
$y = 8$

Equilateral Triangle Corollary If a Δ is equilateral, then it's equiangular.		 each ∠ = 60°
Corollary of the Equiangular Triangle If a Δ is equiangular, then it's equilateral		

Converse of

Example #4 Find each variable or side length.

A. Find x.

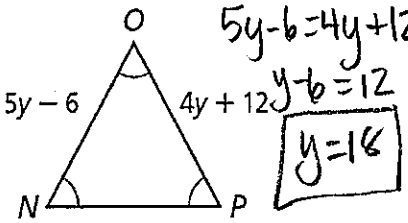


$2x + 32 = 60$

$2x = 28$

$x = 14$

B. Find y.

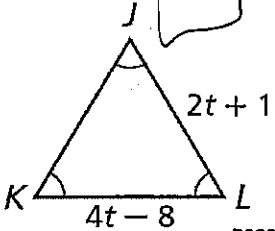


$5y - 6 = 4y + 12$

$y - 6 = 12$

$y = 18$

C. Find JL 10



Chapter 4 Notes

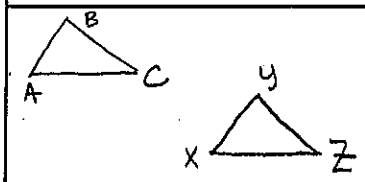
4-3 Congruent Triangles (book 4.3)

Corresponding Angles matching angles > sm to sm

Corresponding Sides matching sides > med. to med, lg to lg

Congruent Polygons exactly the same size & shape

Properties of Congruent Polygons

Diagram	Corresponding Angles	Corresponding Sides
 $\triangle ABC \cong \triangle XYZ$	$\angle A \cong \angle X$ $\angle B \cong \angle Y$ $\angle C \cong \angle Z$	$\overline{AB} \cong \overline{XY}$ $\overline{BC} \cong \overline{YZ}$ $\overline{AC} \cong \overline{XZ}$

* use the △ name to match things up!

Example #1 Naming Congruent Corresponding Parts

* write in corresponding ORDER *

A. Given: $\triangle PQR \cong \triangle STW$

Identify all pairs of congruent corresponding parts.

B. Given: polygon LMNP ≅ polygon EFGH

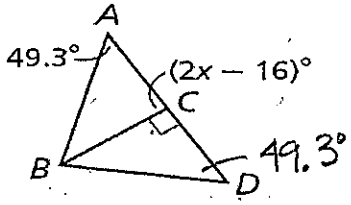
Identify all pairs of congruent corresponding parts.

$\angle L \cong \angle E$, $\angle M \cong \angle F$, $\angle N \cong \angle G$, $\angle P \cong \angle H$

$\overline{LM} \cong \overline{EF}$, $\overline{MN} \cong \overline{FG}$, $\overline{NP} \cong \overline{GH}$, $\overline{LP} \cong \overline{EH}$

Example #2 Using Corresponding Parts of Congruent Triangles

A. $\triangle ABC \cong \triangle DBC$



Find the value of x.

$2x - 16 = 90$

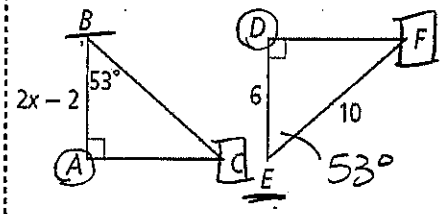
$2x = 106$

$x = 53$

Find m∠DBC.

$90 - 49.3 =$ 40.7°

B. $\triangle ABC \cong \triangle DEF$



Find the value of x.

$2x - 2 = 6$

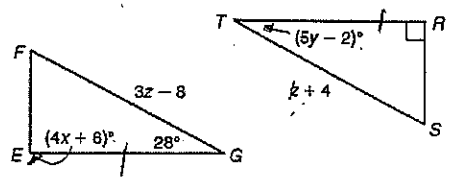
$2x = 8$

$x = 4$

Find m∠F.

$90 - 53 =$ 37°

C. $\triangle EFG \cong \triangle RST$



Find the value of x.

$4x + 6 = 90$

$4x = 84$

$x = 21$

Find $\overline{FG}$.

$3z - 8 = z + 4$

$2z - 8 = 4$

$2z = 12$

$z = 6$

$FG = 3(6) - 8$

$= 10$

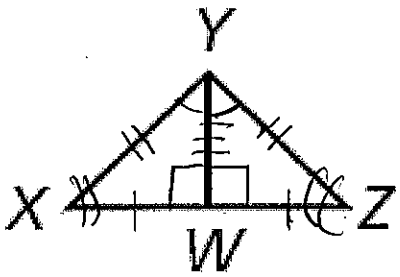
Right Angles Theorem

all rt angles are $\cong$ (RAT)

Example #3 Proving Triangles Congruent

A. Given : $\angle YWX$ and $\angle YWZ$ are right angles.
$\overline{YW}$ bisects $\angle XYZ$.
W is the midpoint of $\overline{XZ}$, $\overline{XY} \cong \overline{YZ}$

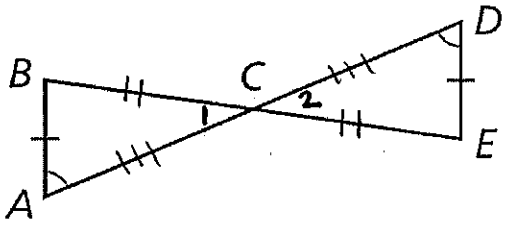
Prove : $\triangle XYW \cong \triangle ZYW$



1. _____	1. Given
2. $\angle YWX \cong \angle YWZ$	2. RAT
3. $\angle XYW \cong \angle ZYW$	3. Def of angle Bisector
4. $\overline{XW} \cong \overline{ZW}$	4. Def of a Mdpt
5. $\overline{YW} \cong \overline{YW}$	5. Reflexive
6. $\angle YXW \cong \angle YZW$	6. Third Angle Thm
7. $\triangle XYW \cong \triangle ZYW$	7. Def of $\cong$ $\triangle$'s

B. Given : $\overline{AD}$ bisects $\overline{BE}$, $\overline{BE}$ bisects $\overline{AD}$,
$\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$

Prove : $\triangle ABC \cong \triangle DEC$



1. _____	1. Given
2. $\overline{BC} \cong \overline{EC}$; $\overline{AC} \cong \overline{DC}$	2. Def of bisector
3. $\angle 1 \cong \angle 2$	3. VAT
4. $\angle B \cong \angle E$	4. Third Angles Thm
5. $\triangle ABC \cong \triangle DEC$	5. Def of $\cong$ $\triangle$'s

Chapter 4 Notes

4.4 Congruent Triangle Theorems: SSS, SAS, HL, ASA, AAS (book 4.4 & 4.5)

Objective: Using short cuts to showing/proving triangles are congruent.

Take all the combinations of sides & angles:

SSS	SAS	SSA
AAA	ASA	AAS

AAA & SSA do not prove $\cong$.

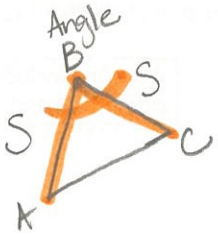
- Question: Does it matter if the A is between the S's or not?

  Or if the S is between the A's or not?

yes, it matters. It's a specific order.

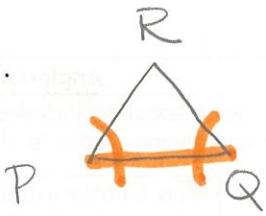
<u>Included Angle</u> - angle formed by 2 adjacent sides.

Example: $\angle B$ is the included angle between $\overline{AB}$ & $\overline{BC}$.



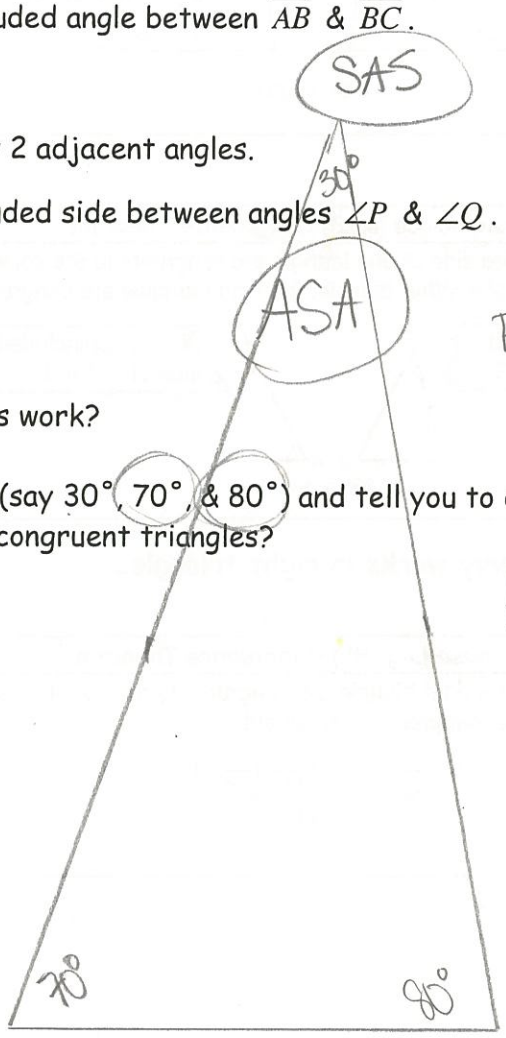
<u>Included Side</u> - side formed by 2 adjacent angles.

Example: $\overline{PQ}$ is the included side between angles $\angle P$ & $\angle Q$.



Question: Do all 6 combinations work?

- If I give you 3 angles (say 30°, 70°, & 80°) and tell you to draw a triangle. Would everyone draw congruent triangles?



AAA does not work to prove $\cong$.

SAS does not work to prove $\cong$.

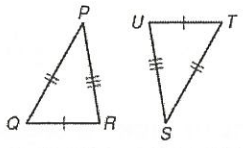
Chapter 4 Notes

Which combinations work?

Side-Side-Side (SSS) Congruence Postulate

If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

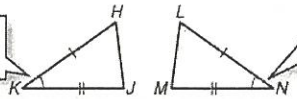
$\overline{QR} \cong \overline{TU}$, $\overline{RP} \cong \overline{US}$, and $\overline{PQ} \cong \overline{ST}$, so $\triangle PQR \cong \triangle STU$.



Side-Angle-Side (SAS) Congruence Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

$\angle K$ is the included angle of $\overline{HK}$ and $\overline{KJ}$.



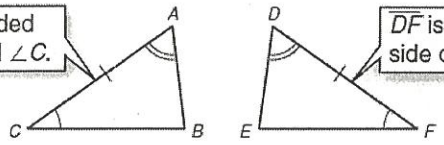
$\angle N$ is the included angle of $\overline{LN}$ and $\overline{NM}$.

$\triangle HJK \cong \triangle LMN$

Angle-Side-Angle (ASA) Congruence Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

$\overline{AC}$ is the included side of $\angle A$ and $\angle C$.



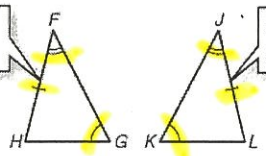
$\overline{DF}$ is the included side of $\angle D$ and $\angle F$.

$\triangle ABC \cong \triangle DEF$

Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a nonincluded side of one triangle are congruent to the corresponding angles and nonincluded side of another triangle, then the triangles are congruent.

$\overline{FH}$ is a nonincluded side of $\angle F$ and $\angle G$.



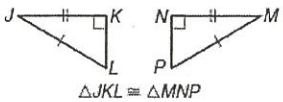
$\overline{JL}$ is a nonincluded side of $\angle J$ and $\angle K$.

$\triangle FGH \cong \triangle JKL$

Special property that only works in right triangles:

Hypotenuse-Leg (HL) Congruence Theorem

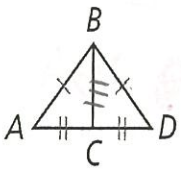
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.



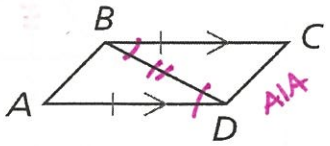
$\triangle JKL \cong \triangle MNP$

Chapter 4 Notes

Example: In #1-12, are the triangles ≅? State the theorem that supports your answer.

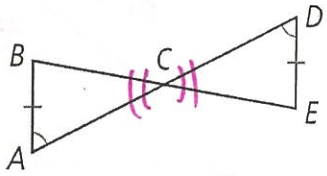
1. 

SSS

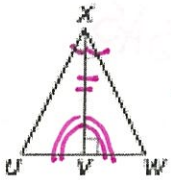
2. 

AIA

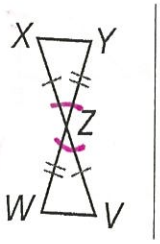
SAS

3. 

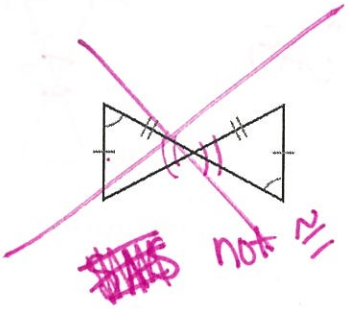
AAS or SAA

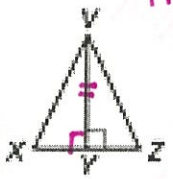
4. 

ASA

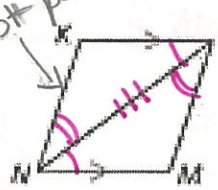
5. 

SAS

6. 
not ≅

7. 

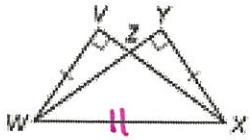
not ≅
not enough info

8. 

not ≅ not parallel

not ≅

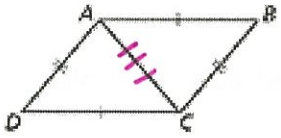
~~ASA~~

9. 

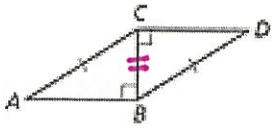
HL

10. 

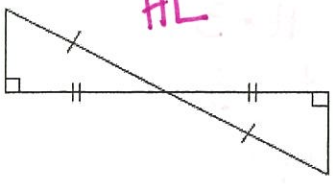
ASA or AAA

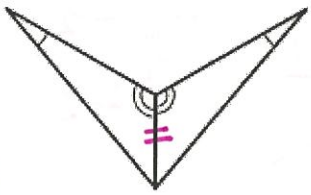
11. 

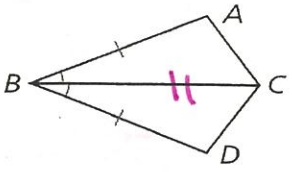
SSS

12. 

HL

13. 
HL

14. 
AAS

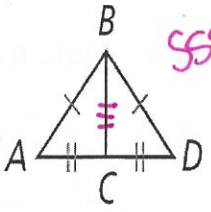
15. 

SAS

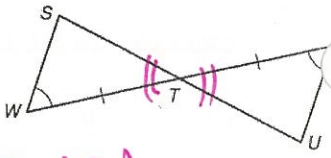
Chapter 4 Notes

16. AAS

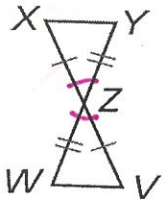
17. SSS



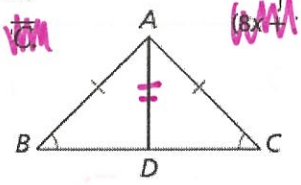
18. ASA



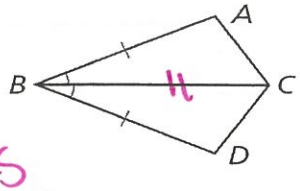
19. SAS



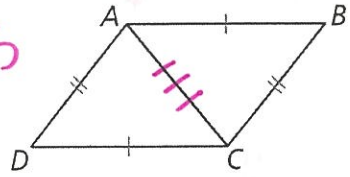
20. ~~VA~~ ~~(8x+1)~~ not ≅



21. SAS

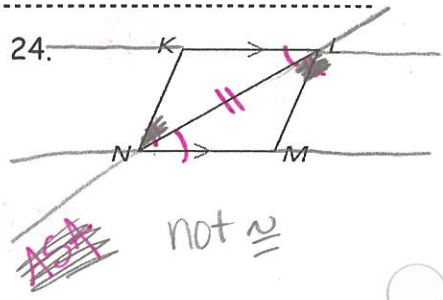


22. SSS

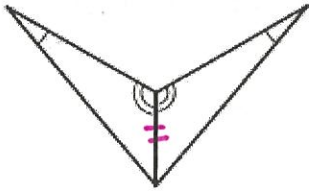


23. not ≅

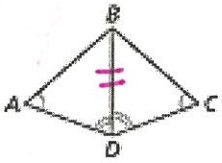
24. not ≅



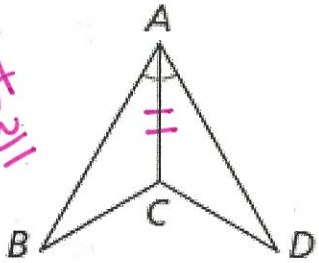
25. AAS



26. AAS



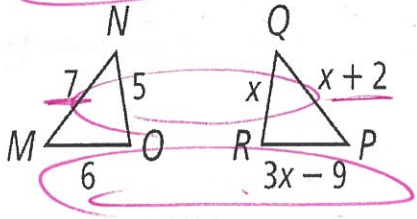
27. not ≅



Examples: In #28 & 29, find each variable.

28. $\Delta PQR \cong \Delta MNO$

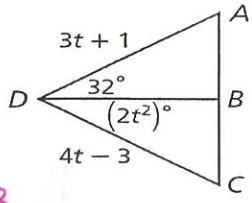
Find x. 5



$x+2=7$
$-2\ \ -2$
$x=5$

29. $\Delta ABD \cong \Delta CBD$

Find t.



$3t+1=4t-3$
$1=t-3$
$t=4$

SSS AAS
SAS
ASA HL

Chapter 4 Notes

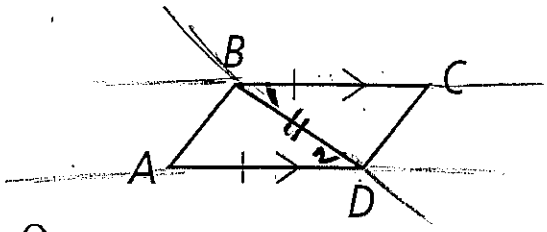
4.5 Proving Triangles are Congruent (book 4.5 and 4.6)

Objective: Completing proofs with congruent triangles.

Examples: Complete the proofs below.

A. Given : $\overline{BC} \parallel \overline{AD}$, $\overline{BC} \cong \overline{AD}$

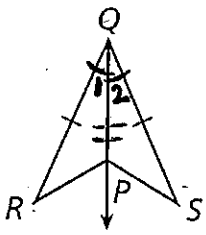
Prove : $\Delta ABD \cong \Delta CDB$



1.	_____	1. Given
2.	$\angle 1 \cong \angle 2$	2. PAIA
3.	$\overline{BD} \cong \overline{BD}$	3. Reflexive
4.	$\Delta ABD \cong \Delta CDB$	4. SAS

B. Given : $\overline{QP}$ bisects $\angle RQS$, $\overline{RQ} \cong \overline{SQ}$

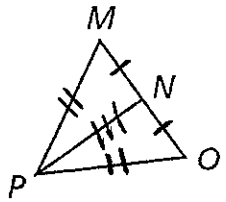
Prove : $\Delta RQP \cong \Delta SQP$



1.	_____	1. Given
2.	$\angle 1 \cong \angle 2$	2. Def of $\angle$ Bisector
3.	$\overline{QP} \cong \overline{QP}$	3. Reflexive
4.	$\Delta RQP \cong \Delta SQP$	4. SAS

C. Given : $\overline{PN}$ bisects $\overline{MO}$, $\overline{MP} \cong \overline{OP}$

Prove : $\Delta MNP \cong \Delta ONP$

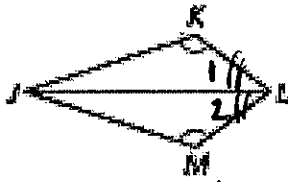


1.	_____	1. Given
2.	$\overline{MN} \cong \overline{ON}$	2. Def. of bisector
3.	$\overline{PN} \cong \overline{PN}$	3. Reflexive
4.	$\Delta MNP \cong \Delta ONP$	4. SSS

Chapter 4 Notes

D. Given : $\overline{JL}$ bisects $\angle KLM$, $\angle K \cong \angle M$

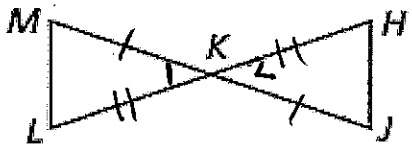
Prove : $\Delta JKL \cong \Delta JML$



1. _____	1. Given
2. $\angle 1 \cong \angle 2$	2. Def of $\angle$ bisector
3. $\overline{JL} \cong \overline{JL}$	3. Reflexive
4. $\Delta JKL \cong \Delta JML$	4. AAS or SAA

E. Given : K is the midpoint of $\overline{HL}$ and $\overline{MJ}$

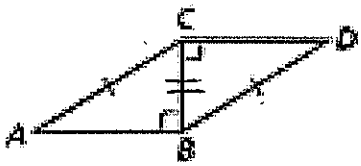
Prove : $\Delta HJK \cong \Delta LMK$



1. _____	1. Given
2. $\overline{MK} \cong \overline{JK}$; $\overline{LK} \cong \overline{HK}$	2. Def of mdpt
3. $\angle 1 \cong \angle 2$	3. VAT
4. $\Delta HJK \cong \Delta LMK$	4. SAS

F. Given : $\overline{AC} \cong \overline{BD}$, $\angle ABC$ and $\angle BCD$ are right angles

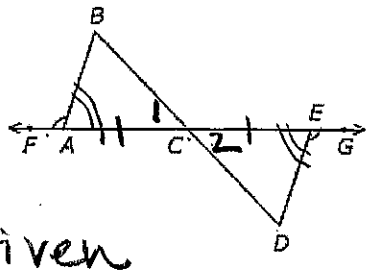
Prove : $\Delta ABC \cong \Delta DCB$



1. _____	1. Given
2. $\angle ABC \cong \angle BCD$	2. Def of Rt angles
3. $\overline{CB} \cong \overline{CB}$	3. Reflexive
4. $\Delta ABC \cong \Delta DCB$	4. HL

Chapter 4 Notes

G. Given : C is the midpoint of $\overline{AE}$, $\angle BAC \cong \angle DEC$



Prove : $\Delta ABC \cong \Delta EDC$

1. _____	1. Given
2. $\overline{AC} \cong \overline{EC}$	2. Def of mdpt
3. $\angle 1 \cong \angle 2$	3. VAT
4. $\Delta ABC \cong \Delta EDC$	4. ASA

4-6 Triangle Congruence CPCF (book 4.6)

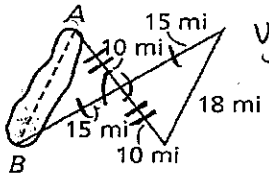
CPCF $\cong$

What does CPCF mean? "Corresponding Parts of Congruent Figures are $\cong$"

Example #1 Find each segment.

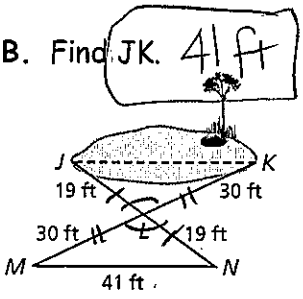
A. Find AB. = 18

have Δ's $\cong$?



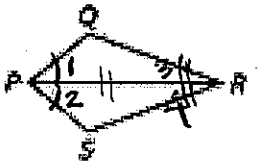
yes by SAS

B. Find JK. 41 ft



SAS $\cong$

Example #2 Fill in each proof.

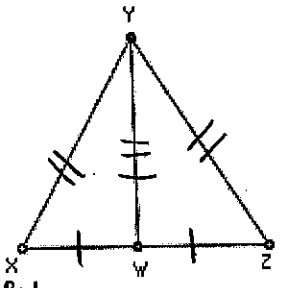


A. Given: $\overline{PR}$ bisects $\angle QPS$ and $\angle QRS$

Prove : $\overline{PQ} \cong \overline{PS}$

1. _____	1. Given
2. $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$	2. Def of $\angle$ bisector
3. $\overline{PR} \cong \overline{PR}$	3. Reflexive
4. $\Delta PQR \cong \Delta PSR$	4. ASA
5. $\overline{PQ} \cong \overline{PS}$	5. CPCF $\cong$

Chapter 4 Notes



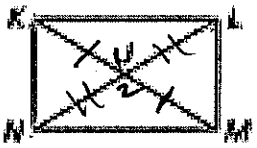
B. Given : $\overline{YW}$ bisects $\overline{XZ}$, $\overline{XY} \cong \overline{YZ}$

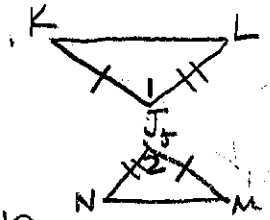
Prove : $\angle XYW \cong \angle ZYW$

1. _____	1. Given
2. $\overline{XW} \cong \overline{ZW}$	2. Def of bisector
3. $\overline{YW} \cong \overline{YW}$	3. Reflexive
4. $\triangle YWX \cong \triangle YWZ$	4. SSS
5. $\angle XYW \cong \angle ZYW$	5. CPCF $\cong$

C. Given : J is the midpoint of $\overline{KM}$ and $\overline{NL}$

Prove : $\angle LKM \cong \angle NMK$

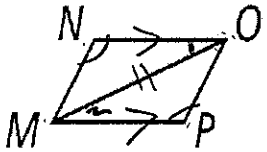




1. _____	1. Given
2. $\overline{KJ} \cong \overline{MJ}$; $\overline{NJ} \cong \overline{LJ}$	2. Def of Mdpt
3. $\angle 1 \cong \angle 2$	3. VAT
4. $\triangle KJL \cong \triangle MJN$	4. SAS
5. $\angle LKM \cong \angle NMK$	5. CPCF $\cong$

D. Given : $\overline{NO} \parallel \overline{MP}$, $\angle N \cong \angle P$

Prove : $\overline{PO} \cong \overline{MN}$



1. _____	1. Given
2. $\angle 1 \cong \angle 2$	2. PAIA
3. $\overline{MO} \cong \overline{MO}$	3. Reflexive
4. $\triangle POM \cong \triangle MNO$	4. AAS
5. $\overline{PO} \cong \overline{MN}$	5. CPCF $\cong$