Classroom Copy

Chapter 6 Lesson #1

Objective: Simplifying integer exponents (book 7-1) → exponent

What is an exponent? x^y → base

What does 3^2 mean? $3 \cdot 3 = 9$

Zero Exponents

$x^0 = 1$

$7^0 = 1$ $\quad (-5)^0 = 1$ $\quad 10^0 = 1$

the opposite of 5^0

$-5^0 = -1$

$(-5)^0 = 1$ $\quad (-5)^0$

Negative Exponents

opposite

$x^{-n} = \frac{1}{x^n}$

* take Reciprocal of base
* then exponent is positive
* calculate with positive exponent
* Keep as a fraction

$x^{-5} = \left(\frac{1}{x}\right)^5 = \frac{1}{x^5}$

$2^{-4} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

$\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$

Examples: Simplify with positive exponents and without decimals.

1. 2^{-3} $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$
2. 5^{-2} $\left(\frac{1}{5}\right)^2 = \frac{1}{25}$
3. $(-3)^{-4}$ $\left(\frac{1}{-3}\right)^4 = \frac{1}{81}$
4. $(-4)^0$ 1
5. 5^0 1
6. 7^{-2} $\frac{1}{49}$
7. 6^{-3} $\left(\frac{1}{6}\right)^3 = \frac{1}{216}$
8. -6^0 -1

What if the negative exponent is in the denominator?

Examples:

$\frac{1}{k^{-2}} = \frac{k^2}{1} = k^2$

$\frac{1}{x^{-6}} = x^6$

Examples: Simplify with positive exponents and without decimals.

1. $\frac{5}{m^{-3}} = 5m^3$

2. $\frac{10}{2^{-3}}$

$\frac{10}{1} \cdot \frac{1}{2^{-3}} = \frac{10}{1} \cdot \frac{2^3}{1} =$

$10 \cdot 8 = 80$

3. $\frac{x^3}{y^{-2}z}$ $\frac{x^3y^2}{z}$

4. $\frac{6m^{-4}}{2n^{-1}p^6}$ $\frac{3n}{1m^4p^6}$ $\frac{3n}{m^4p^6}$

5. $\frac{9w^0r^{-7}}{t^{-4}} = \frac{9t^4}{r^7}$

6. $\frac{6w^{-2}x^{-3}}{z^0y^{-4}}$ $\frac{6y^4}{w^2x^3}$

7. $\frac{6x^{-3}y^4}{m^{-6}n^{-1}}$ $\frac{6m^6ny^4}{x^3}$

8. $\frac{5x^{-2}y^{-3}}{10a^5b^{-4}}$ $\frac{b^4}{2a^5x^2y^3}$

9. $\frac{3f^0}{5g^{-8}}$ $\frac{3g^8}{5}$

10. $\frac{2m^{-7}}{p^{-5}}$ $\frac{2p^5}{m^7}$

11. $\frac{m^{-4}n^0}{6p^2}$ $\frac{1}{6m^4p^2}$

12. $3m^{-5}$ $\frac{3}{m^5}$

Chapter 6 Lesson #2

Objective: Simplifying integer exponents (book 7-2 & 7-3)

Rules to Simplifying Expression

1. NO negative exponents
2. Same base does not appear more than once
3. no powers are raised to powers
4. no products are raised to powers
5. no quotients are raised to powers
6. Coefficients simplified (Reduce fractions)

Product of Powers (same base)	$x^a \cdot x^b = x^{a+b}$ $\quad x^2 \cdot x^6 = x^{2+6} = x^8$ $\quad 5 \cdot 5^7 = 5^8$
Power of a Product (same base)	$(2x^3)^4 = 2^{1\cdot4}\, x^{3\cdot4} = 2^4 x^{12} = \boxed{16x^{12}}$ $\quad (xy)^3 = x^3y^3$ $\quad (x^2y)^3 = x^6y^3$
Power of a Power (Same base)	$(x^a)^b = x^{ab}$ $\quad (x^3)^3 = (x^3)(x^3)(x^3) = \boxed{x^9}$ $\quad (7^2)^4 = 7^8 = 5,764,801$

Examples: Simplify

1. $m^5 \cdot y^3 \cdot m^3$ $\quad \boxed{m^8y^3}$

2. $y^3 \cdot z^6 \cdot y^4$ $\quad \boxed{y^7z^6}$

3. $(x^2)^{-3} \cdot x^4$ $\quad x^{-6} \cdot x^4 = x^{-2} = \boxed{\frac{1}{x^2}}$

4. $(m^{-3})^4 \cdot m^6$ $\quad m^{-12} \cdot m^6 = m^{-6} = \boxed{\frac{1}{m^6}}$

5. $3^4a^4b^5a^2$ $\quad \boxed{81a^6b^5}$

6. $(x^7)^0 \cdot (-2x^3)^4$ $\quad 1 \cdot (-2)^4(x^3)^4$ $\quad \boxed{16x^{12}}$

7. $x^5 \cdot y^2 \cdot x^{-2}$ $\quad \boxed{x^3y^2}$

8. $3x^2 \cdot (y^5)^3 \cdot m^{-2}$ $\quad 3x^2 \cdot y^{15} \cdot m^{-2} = \boxed{\frac{3x^2y^{15}}{m^2}}$

9. $\left(x^{-6}y^{4}\right)^{-3}\cdot x^{5}$

$x^{18}y^{-12}x^{5}=$ $\dfrac{x^{23}}{y^{12}}$

10. $\left(x^{-5}y^{2}\right)^{3}\cdot x^{8}$

$x^{-15}y^{6}\cdot x^{8}=x^{-7}y^{6}=$ $\dfrac{y^{6}}{x^{7}}$

11. $\left(5m^{3}\right)^{2}\left(m^{2}\right)^{4}$

$25m^{6}\cdot m^{8}=$ $25m^{14}$

12. $\left(x^{2}y^{3}\right)^{4}\cdot\left(x^{2}y^{4}\right)^{-4}$

$x^{8}y^{12}\cdot x^{-8}y^{-16}=y^{-4}=$ $1/y^{4}$

Quotient of Powers (Same base)

$\dfrac{x^{m}}{x^{n}}=x^{(m-n)}$

$\dfrac{x^{5}}{x^{2}}=x^{3}$

$\dfrac{x^{2}}{x^{10}}=x^{-8}=\dfrac{1}{x^{8}}$

Power of a Quotient (Same base)

$\left(\dfrac{x^{3}}{x^{4}}\right)^{5}=\left(x^{3-4}\right)^{5}=\left(x^{-1}\right)^{5}=x^{-5}=\dfrac{1}{x^{5}}$

$\left(\dfrac{3}{5}\right)^{3}=\dfrac{3^{3}}{5^{3}}=\dfrac{27}{125}$

Examples: #1-6, simplify each expression.

1. $\dfrac{2^{9}}{2^{7}}=2^{2}=4$

2. $\dfrac{x^{5}}{x^{5}}=x^{0}=1$

3. $\dfrac{20m^{5}n^{4}}{4m^{3}n^{8}}$ $\dfrac{5m^{2}}{n^{4}}$

4. $\dfrac{a^{5}b^{9}}{(ab)^{4}}=\dfrac{a^{5}b^{9}}{a^{4}b^{4}}=$ ab^{5}

5. $\dfrac{p^{6}q^{12}}{(pq)^{4}}$

$\dfrac{p^{6}q^{12}}{p^{4}q^{4}}=p^{2}q^{8}$

6. $\dfrac{m^{7}n^{3}}{(mn)^{2}}$

$\dfrac{m^{7}n^{3}}{m^{2}n^{2}}=m^{5}n$

7. $\dfrac{2xy^{-3}}{3x^{-5}y}$ $x^{1-(-5)}\,y^{-3-1}$

$\dfrac{2}{3}x^{6}y^{-4}=\dfrac{2x^{6}}{3y^{4}}$

8. $\dfrac{2^{3}\cdot3^{2}\cdot5^{7}}{2\cdot3^{4}\cdot5^{5}}$

$\dfrac{2^{2}\cdot5^{2}}{3^{2}}=\dfrac{4\cdot25}{9}$ $\dfrac{100}{9}$

9. $\dfrac{x^{-2}p^{4}}{(xp)^{3}}$

$\dfrac{x^{-2}p^{4}}{x^{3}p^{3}}=x^{-5}p^{1}=$ $\dfrac{p}{x^{5}}$

10. $\dfrac{10x^{-2}y^{5}}{20x^{5}y^{2}}$

$\dfrac{1}{2}x^{-7}y^{3}=$ $\dfrac{y^{3}}{2x^{7}}$

11. $\dfrac{3x^{3}y^{5}}{12x^{-7}y^{-2}}$

$\dfrac{1}{4}x^{10}y^{7}$ OR $\dfrac{x^{10}y^{7}}{4}$

12. $\left(\dfrac{5ab^{4}}{3a^{5}b}\right)^{2}$

$\dfrac{5^{2}a^{2}b^{8}}{3^{2}a^{10}b^{2}}=\dfrac{25b^{6}}{9a^{8}}$

$\left[\dfrac{5b^{3}}{3a^{4}}\right]^{2}=\dfrac{5^{2}b^{6}}{3^{2}a^{8}}=$

Scientific notation

Chapter 6 Lesson #3

Objective: Simplifying integer exponents (book 7-2, 7-5, & 7-6)

Scientific Notation - $\square \times 10^{\square}$ ← # ; $\square$ → #1-10

Property	Numbers	Calculator
Positive Integer Exponent 23,000,000	2.3×10^{7}	2.3 E 7 2.3^{7}
Negative Integer Exponent .00000045	4.5×10^{-7}	4.5 E -7

Examples: #1-4, write as a power of 10.

1. 10,000 ~~10×10^{4}~~ 1×10^{4}
2. 100 1×10^{2}
3. 0.001 1×10^{-3}
4. 0.1 1×10^{-1}

Examples: #5-8, write the following numbers in scientific notation.

5. 97,600,000 9.76×10^{7}
6. 0.547
7. 45,780 4.578×10^{4}
8. 0.00195

9. 729,000,000
10. 0.00000456 4.56×10^{-6}
11. 312,000,000,000

Examples: #12 - 15, find the value of each expression.

12. 1.95×10^{-3} .00195
13. 8.534×10^{5} 853,400
14. 0.163×10^{-2}
15. 12.5×10^{-6}

16. Saturn has a diameter of about 1.2×10^{5} km. Its distance from the Sun is about 1,427,000,000 km.

    (a) Write Saturn's diameter in standard form. 120000 km

    (b) Write Saturn's distance from the sun in scientific notation. 1.427×10^{9}

Monomial - one term; Must have whole # exponents

Example: 5; x; $7xy$; $\frac{1}{2}x^2$ { non-examples: $-3x^{1/2}$; $\frac{2}{x^3}$; $4x-y$ $\rightarrow 2x^{-3}$

Degree - Sum of the exponents of the variables

* the degree of a constant is zero!

Examples: State the degree of each polynomial.

1. $5x$ [1]
2. $10y^2$ [2]
3. $6x^3y^2 - 2y$ [5]
4. $6x^2y^5 + 4xy + y^2$ [7]
5. $10y^2$ [3]
6. $4xy^5$ [6]
7. $6y^2 - 3x^5y^3$ [8]
8. $3xy^4 + 6x^4y^2 + 4x^3$ [6]

Standard Form - when the polynomial is written from highest to lowest degree (w/ one variable)

Term - $3x^2 + 6x - 8$ (separated by + or - sign)

Examples: Write each polynomial in standard form.

9. $4x - 5x^3 + 2 + 5x^2$
$-5x^3 + 5x^2 + 4x + 2$

10. $5xy + 7x^3 - 9y^2$
$7x^3 + 5xy - 9y^2$

11. $10a - 3b^2 + 5a^2b + 2ab + 3$
$5a^2b + 2ab + 10a - 3b^2 + 3$

Some polynomials have special names based on their degree and the number of terms they have.

Degree	Name
0	Constant
1	Linear $y = mx + b$
2	Quadratic x^2
3	Cubic x^3
4	Quartic x^4
5	Quintic x^5
6 or more	6th degree, 7th degree, and so on

Terms	Name
1	Monomial
2	Binomial
3	Trinomial
4 or more	Polynomial

Examples: Classify each polynomial by its degree and number of terms.

12. $5x - 6$
Linear binomial

13. $y^3 + y + 4$
Cubic trinomial

14. $3x^4 - 3x - 6$
Quartic trinomial

15. $6x + 9x^2 - x + 3$
Quadratic polynomial

Chapter 6 Lesson #4

Objective: Adding and Subtracting Polynomials (book 7-4)

Review : Simplify $+ \not{=} -$ only with like terms

1. $4x + 2x$ $= 6x$

2. $5p - 6p^2$

Examples : Simplify the following polynomials and write your answer in standard form.

3. $20x^2 + 7x - 3x^2$

$17x^2 + 7x$

4. $4m^4 - 8 + 16m^4 + 2$

$20m^4 - 6$

5. $(4y^2 + 5) + (y^2 - y + 6)$

$5y^2 - y + 11$

6. $(10xy + x) - (-3xy - y)$

$10xy + x + 3xy + y$

$x + 13xy + y$

7. $2(5a^3 + 3a^2 - 6a + 12a^2) + (7a^2 - 10a)$

$10a^3 + 6a^2 - 12a + 24a^2 + 7a^2 - 10a$

$10a^3 + 37a^2 - 22a$

8. $(x^2 - 9) - (10x^2 - 3x + 7)$

$x^2 - 9 - 10x^2 + 3x - 7$

$-9x^2 + 3x - 16$

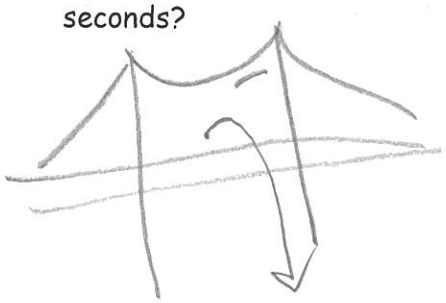
9. $(t^4 - 2t) - (3t^4 - 3t^2 + 1)$

$t^4 - 2t - 3t^4 + 3t^2 - 1$

$-2t^4 + 3t^2 - 2t - 1$

Application

10. A tourist accidentally dropped his keys off the Golden Gate Bridge. The bridge is 220 feet from the water of the bay. The height of the keys in feet is given by the polynomial $-16t^2 + 220$, where t is time in seconds. How far above the water will the keys be after 3 seconds?



$-16t^2 + 220$

$-16(3)^2 + 220$

$-16(9) + 220$

76 ft

Chapter 6 Lesson #5

Objective: Simplifying integer exponents (book 7-2, 7-5, & 7-6)

Part I: Multiplying Monomials

Examples: Simplify

1. $(2x)(7x^3)$

$14x^4$

2. $(3m^2n)(5mn^3)(-m^5n^2)$

$-15m^8n^6$

3. $(\frac{1}{4}x^2y)(12x^3z^2)(y^4z^3)$

$3x^5y^5z^5$

Part II: Multiplying a Polynomial by a Monomial

Examples: Simplify the following polynomials & write your answer in standard form.

4. $5x(3x^2 + 4x - 7)$

$15x^3 + 20x^2 - 35x$

5. $4r^2s^2(r - 3s)$

$4r^3s^2 - 12r^2s^3$

Part III: Multiplying a Binomial by a Binomial

2 Methods: (1) FOIL or (2) distributing

F (first terms) O (outer terms) I (inner terms) L (last terms)

Examples: Simplify the following polynomials & write your answer in standard form.

6. $(x + 2)(x + 4)$

$x^2 + 4x + 2x + 8$

$x^2 + 6x + 8$

7. $(y - 4)(y + 6)$

$y^2 + 6y - 4y - 24$

$y^2 + 2y - 24$

8. $(x + 2)(x + 2)$ $\rightarrow (x+2)^2$

$x^2 + 2x + 2x + 4$

$x^2 + 4x + 4$

9. $(y - 4)(y + 4)$

$y^2 + 4y - 4y - 16$

$y^2 - 16$

10. $(m - 2)(m + 2)$

$m^2 + 2m - 2m - 4$

$m^2 - 4$

11. $(y - 6)(y + 6)$

$y^2 - 36$

12. $(p + 8q)(p - 8q)$

$p^2 - 8pq + 8pq - 64q^2$

$p^2 - 64q^2$

13. $(2n - 7)(2n + 7)$

$4n^2 + 14n - 14n - 49$

$4n^2 - 49$

14. $(y + 7)^2$

$(y+7)(y+7)$

$y^2 + 7y + 7y + 49$

$y^2 + 14y + 49$

15. $(x + 5)^2$

$(x+5)(x+5)$

$x^2 + 5x + 5x + 25$

$x^2 + 10x + 25$

16. $(2x + 6)^2$

$(2x+6)(2x+6)$

$4x^2 + 12x + 12x + 36$

$4x^2 + 24x + 36$

17. $(4r - 3)^2$

$(4r-3)(4r-3)$

$16r^2 - 12r - 12r + 9$

$16r^2 - 24r + 9$

Part IV: Multiplying any size polynomial

18. $(x - 5)(x^2 + 4x - 6)$

$x^3 + 4x^2 - 6x - 5x^2 - 20x + 30$

$x^3 - x^2 - 26x + 30$

19. $(3x + 1)(x^3 + 4x^2 - 7)$

$3x^4 + 12x^3 - 21x + x^3 + 4x^2 - 7$

$3x^4 + 13x^3 + 4x^2 - 21x - 7$

Property	Algebra	Numbers
zero exponent	$x^0 = 1$	$5^0 = 1$
negative exponent	$x^{-y} = \frac{1}{x^y}$	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
product of powers	$x^y \cdot x^z = x^{y+z}$	$2^3 \cdot 2^5 = 2^{3+5} = 2^8 = 256$
power of a power	$(x^y)^z = x^{yz}$	$(2^3)^4 = 2^{3 \cdot 4} = 2^{12} = 4096$
power of a product	$(xy)^z = x^z \cdot y^z$	$(4 \cdot 3)^2 = 4^2 \cdot 3^2 = 16 \cdot 9 = 144$
quotient of powers	$\left(\frac{x^y}{x^z}\right) = x^{y-z}$	$\left(\frac{2^5}{2^3}\right) = 2^{5-3} = 2^2 = 4$
power of a quotient	$\left(\frac{x}{y}\right)^z = \frac{x^z}{y^z}$	$\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$

$(x+y)^2 = x^2 + 2xy + y^2$

$(x-y)^2 = x^2 - 2xy + y^2$

$(x+y)(x-y) = x^2 - y^2$