Key

CH 12 Notes

12-1 Reflections (Book 12-1)

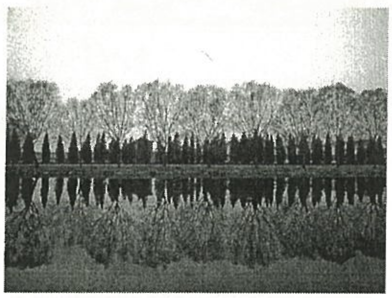
Objective: Understand what an isometry is and be able to draw a reflection.

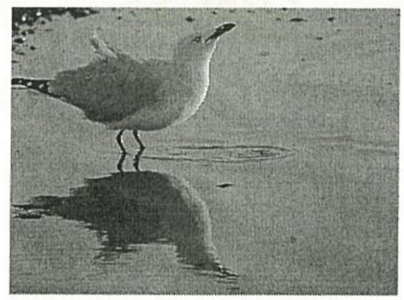
Isometry (or transformation) - movement that does NOT change the shape or size of a figure.

4 types:

1. Reflection
2. Rotations
3. Translation
4. Glide Reflections

Today we're going to look at just one of the four isometries, **REFLECTIONS**.

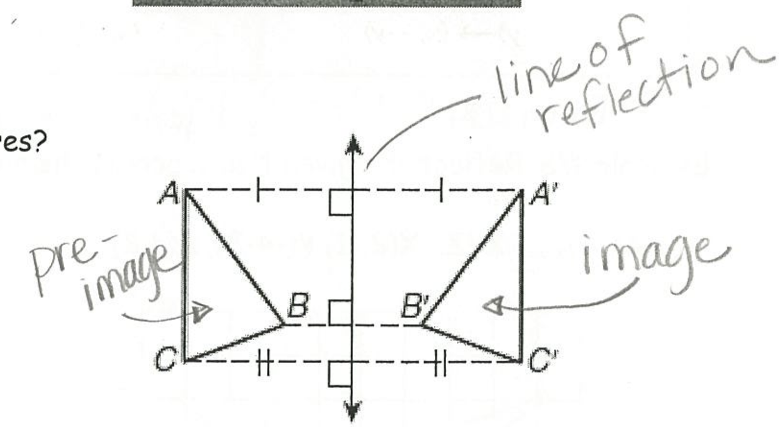




Class Example:

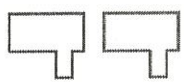
- What do you notice about the two figures?

If you connect the pre-image and image, the Reflecting line is the ⊥ bisector of that connecting segment.



A **reflection** is a transformation across a line, called the line of reflection, so that it's the perpendicular bisector of each segment joining each point and its image.

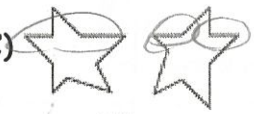
Example #1 Tell whether each transformation appears to be a reflection.

A) 

NO

B) 

yes

C) 

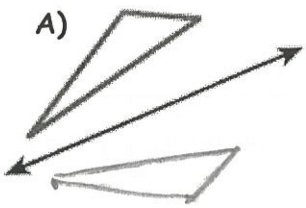
NO

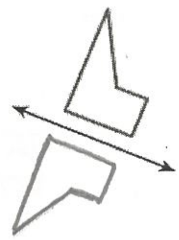
D) 

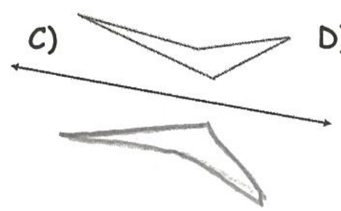
yes

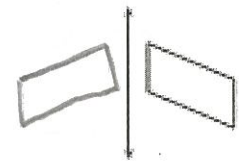
Example #2 Draw the reflection of the figures across the line.

use tracing paper

A) 

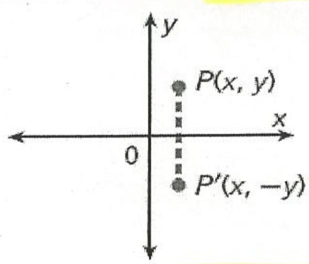
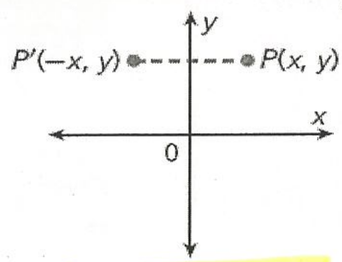
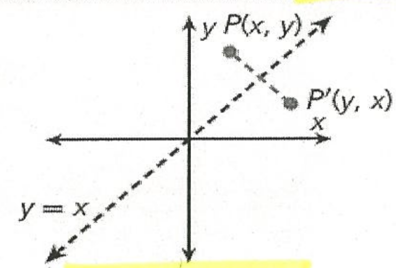
B) 

C) 

D) 

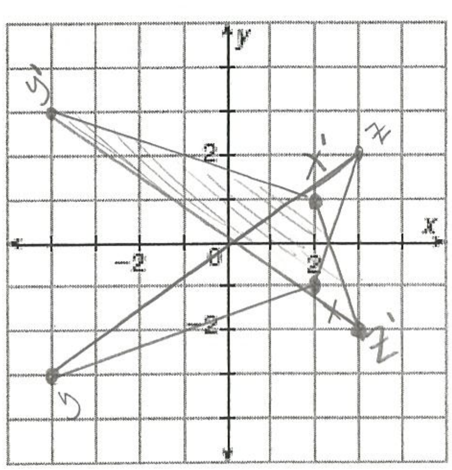
Reflections in the Coordinate Plane

reflection notation

ACROSS THE *x*-AXIS	ACROSS THE *y*-AXIS	ACROSS THE LINE $y = x$
		
$(x, y) \longrightarrow (x, -y)$	$(x, y) \longrightarrow (-x, y)$	$(x, y) \longrightarrow (y, x)$
$r_{x\,axis}(x,y) \rightarrow (x,-y)$	$r_{y\,axis}(x,y) \rightarrow (-x,y)$	$r_{y=x}(x,y) \rightarrow (y,x)$

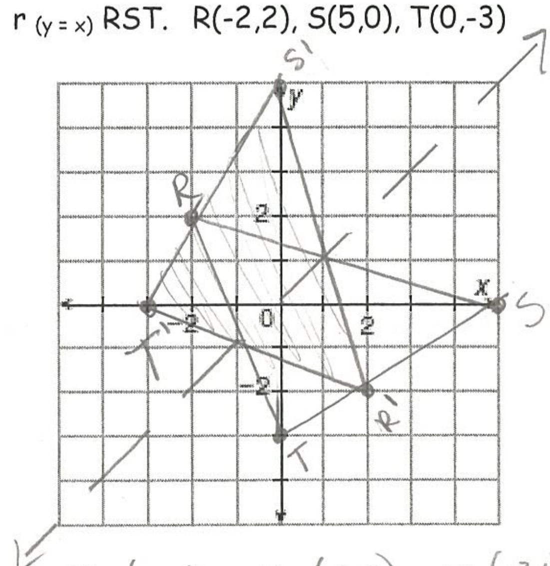
Example #3 Reflect the given figure across the given line.

A) $r_{(x\text{ axis})}XYZ$. X(2,-1) Y(-4-3), Z(3,2)



X′ = (2,1)   Y′ = (-4,3)   Z′ = (3,-2)

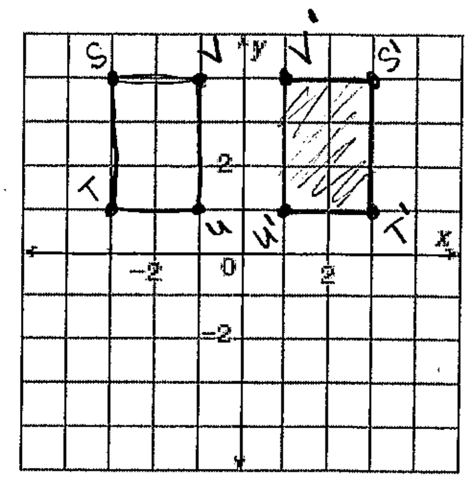
B) $r_{(y = x)}$ RST. R(-2,2), S(5,0), T(0,-3)



R′ = (2,-2)   S′ = (0,5)   T′ = (-3,0)

C) $r_{(y\text{-axis})}$ $S(-3,4), T(-3,1), U(-1,1), V(-1,4)$

$S'(3,4)$ $T'(3,1)$ $U'(1,1)$ $V'(1,4)$



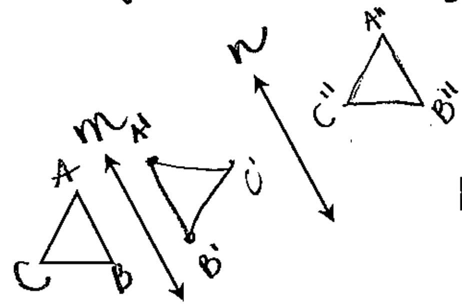
12-2 Translations (Book 12.2)

Objective: Understanding translations.

Class Example:

1st: Draw $r_m(\Delta ABC) = A'B'C'$

2nd: Draw $r_n(\Delta A'B'C') = A''B''C''$



Reflecting over 2 // lines = a translation (slide).

This is a translation!

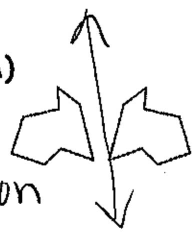
A **translation** is an isometry where all the points of a figure are moved the same distance in the same direction.

Because you performed 2 reflections consecutively, we can write the notation a little different:

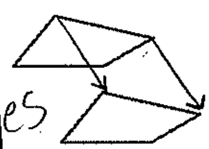
$r_n \circ r_m(\Delta ABC)$ or $r_n(r_m(\Delta ABC))$

read bkwards

Example #1 Tell whether each transformation appears to be a translation. Explain.

A) 

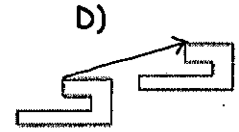
no reflection

B) 

yes

C) 

no

D) 

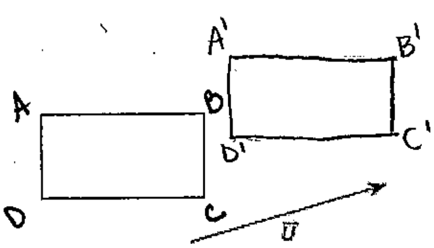
yes

There is another way to perform a translation...place your tracing paper over top of ΔABC in the class example above. Trace ΔABC onto your tracing paper. Now slide your paper towards $\Delta A''B''C''$.

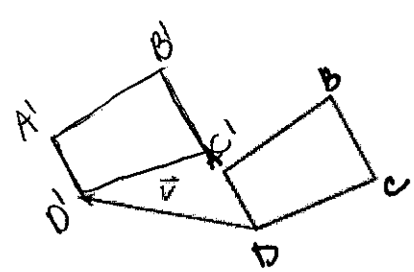
The line that you slide along is called a vector. Vectors have direction & magnitude (size)

Example #2 Drawing Translations

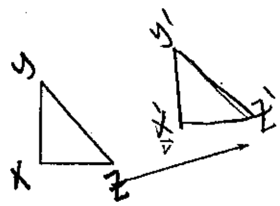
A)



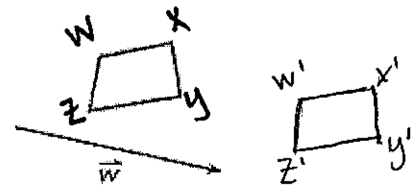
B)



C)



D)



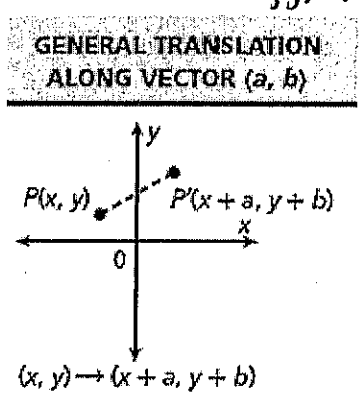
Properties of Translations:

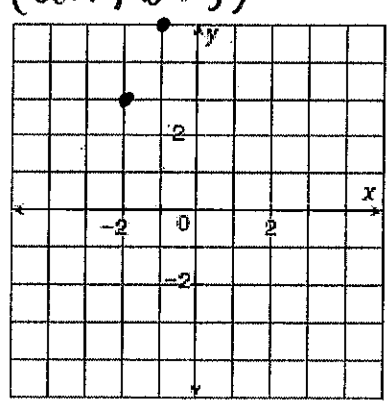
1. A translation is <u>2</u> reflections over <u>parallel</u> lines.
2. A translations magnitude is <u>twice</u> the distance between the parallel lines.

Translations in the Coordinate plane:

When dealing with a coordinate plane, translations are very simple. The vector is given as an ordered pair. For example $\vec{v} = \langle 1,2 \rangle$, means that the pre-image moves 1 unit to the right and up 2 units.

$T_{\langle x,y \rangle}(a,b) = (a+x, b+y)$





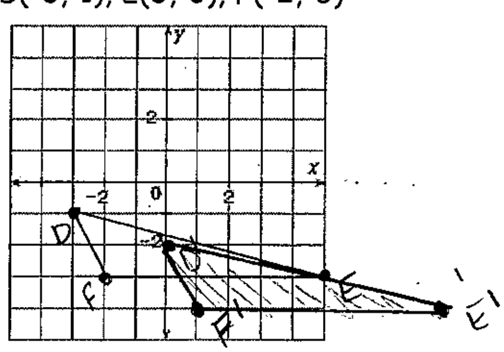
$T_{\langle 1,2 \rangle}(-2,3)$

$= (-1,5)$

Example #3 Drawing Translations in the Coordinate Plane

A) $T_{\langle 3,-1 \rangle} \triangle DEF$

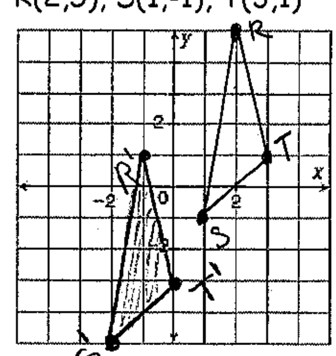
D(-3,-1), E(5,-3), F(-2,-3)



D'(0,-2) E'(8,-4) F'(1,-4)

B) $T_{\langle -3,-4 \rangle} \triangle RST$

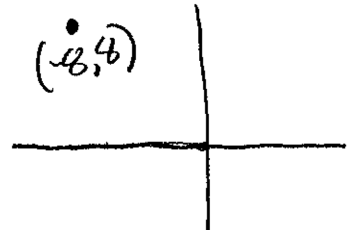
R(2,5), S(1,-1), T(3,1)



R'(-1,1) S'(-2,-5) T'(0,-3)

Applications:

Examples: In #4 & 5, complete the given problem.



4. In a marching band drill, a drummer starts 8 steps to the left and 8 steps up from the center of the field. She then marches 16 steps to her right to her second position. Finally she marches 24 steps backwards to her last position.

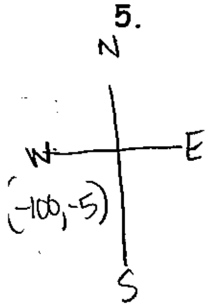
(a) What is her final position? $T_{\langle 16,-24\rangle}(-8,8) = \boxed{(8,-16)}$

(b) Give the vector that describes her moves.

$\langle 16,-24\rangle$

5. A sailboat has coordinates 100° west and 5° south. The boat sails 50° due west. Then the boat sails 10° due south.

−50 −10



(a) What is the boat's final position? $(-100,-5) + \langle -50,-10\rangle = \boxed{(-150,-15)}$

(b) Give the vector that describes the move.

$\boxed{\langle -50,-10\rangle}$

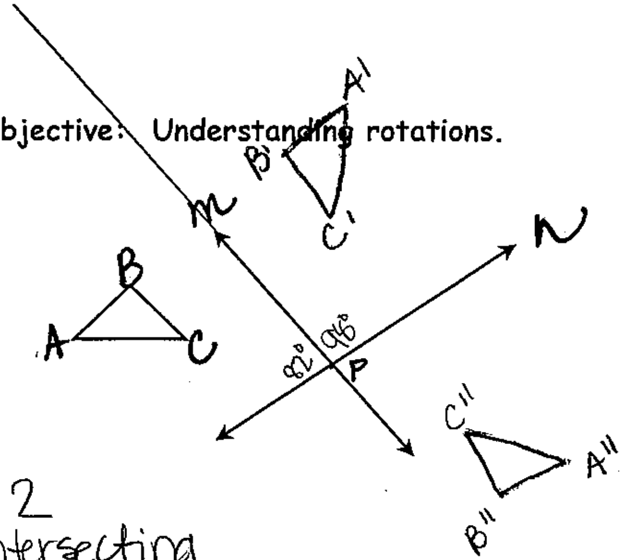
12-3 Rotations (Book 12.3)

Objective: Understanding rotations.

Class Example:

2nd 1st

Draw $r_n(r_m(\Delta ABC)) = A''B''C''$



Questions:

(1) How many reflections did you perform? 2

(2) Are the lines parallel? Intersecting? intersecting

This is a rotation!

Explore:

(1) Place tracing paper over ΔABC & trace ΔABC.

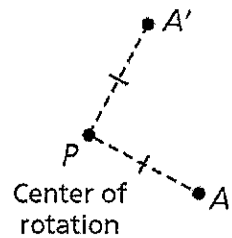
(2) Put your pencil on P.

(3) Spin your paper until your preimage is on top of your image.

* Rotations can be called a ________________ or ____________________.

Rotations

A rotation is a transformation about a point *P*, called the center of rotation, such that each point and its image are the same distance from *P*, and such that all angles with vertex *P* formed by a point and its image are congruent. In the figure, $\angle APA'$ is the angle of rotation.



Example #1 Identify whether or not each transformation is a rotation.

A)

NO (translation)

B)

yes, Rotation

C)

yes, Rotation

D)

NO, Reflection

There is another way to perform rotations besides doing 2 reflections.

Go back to the class example:

(1) Measure the acute angle between lines m & n.

(2) Measure the angle between A & A".

(3) What do you notice?

Since rotations are like spinning or turning, we often talk about the direction of the turn. This direction is referred to as ___clockwise___ or ___counterclockwise___.

___Clockwise___ is in the ___negative___ direction.

___Counterclockwise___ is in the ___positive___ direction.

* think Quandrant order QII | QI / QIII | QIV

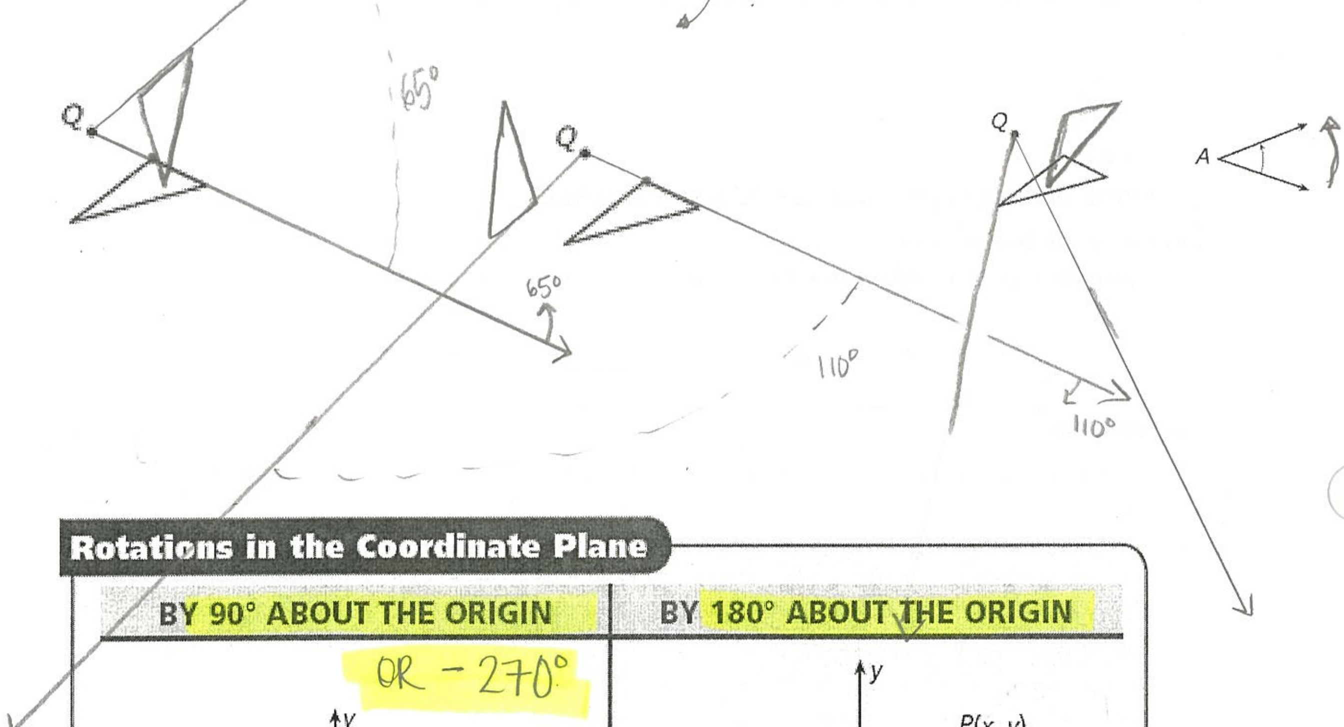
Examples: In #2-4, complete the given rotation about point Q.

*

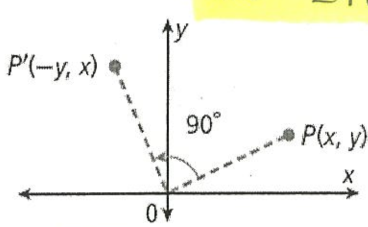
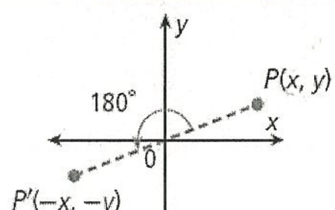
2. Rotation of 65°

3. Rotation of -110°

4.



Rotations in the Coordinate Plane

BY 90° ABOUT THE ORIGIN	BY 180° ABOUT THE ORIGIN
OR −270°  $(x, y) \rightarrow (-y, x)$	 $(x, y) \rightarrow (-x, -y)$

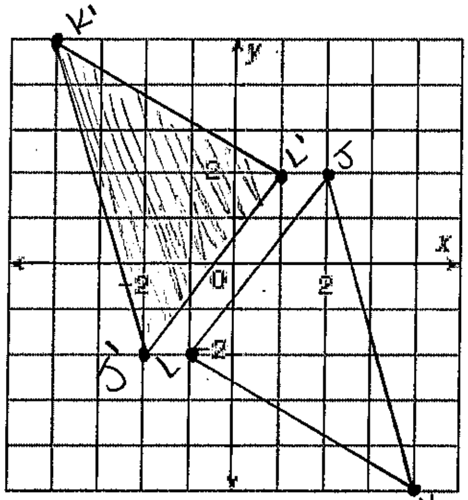
$R_{90}(x,y) = (-y, x)$ $R_{180}(x,y) = (-x,-y)$ $R_{270}(x,y) = (y,-x)$

or $R_{-270}(x,y) \rightarrow (-y,x)$ or $R_{-180}(x,y) \rightarrow (-x,-y)$ or $R_{-90}(x,y) \rightarrow (y,-x)$

Example #5 Drawing Rotations in the Coordinate Plane

A) $R_{180}\ \triangle JKL$ $(-x,-y)$

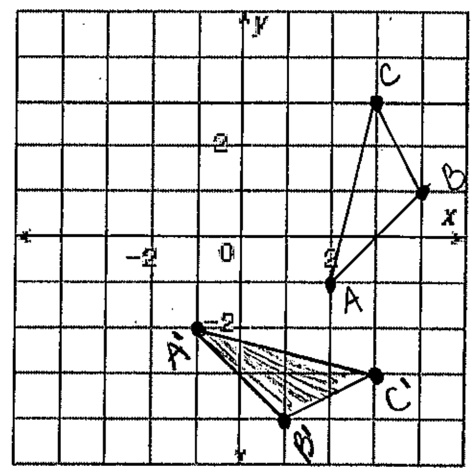
$J(2,2)$, $K(4,-5)$, $L(-1,-2)$



J' $(-2,-2)$ K' $(-4,5)$ L' $(1,2)$

B) $R_{270°}\ \triangle ABC$ $(y,-x)$

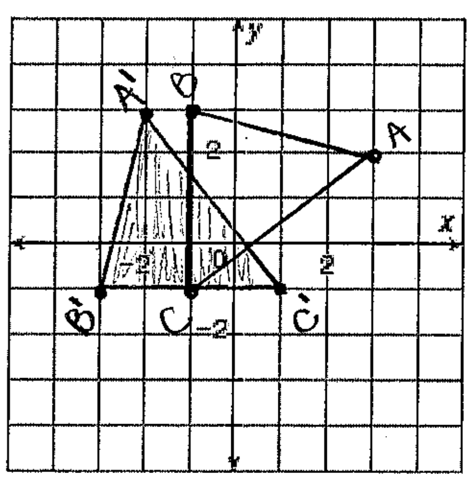
$A(2,-1)$, $B(4,1)$, $C(3,3)$



A' $(-1,-2)$ B' $(1,-4)$ C' $(3,-3)$

C) $R_{90°}\ \triangle ABC$ $(-y,x)$

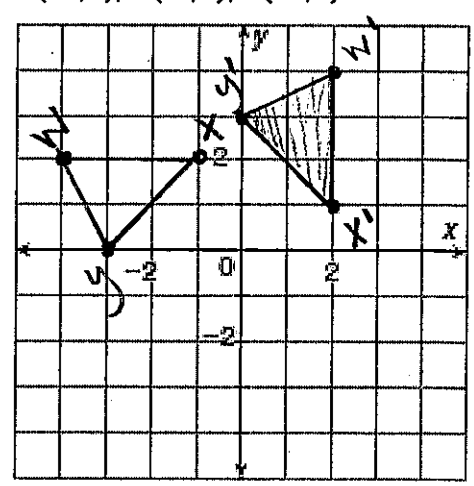
$A(3,2)$, $B(-1,3)$, $C(-1,-1)$



A' $(-2,3)$ B' $(-3,-1)$ C' $(1,-1)$

D) $R_{-90°}\ \triangle WXY$ $(y,-x)$

$W(-4,2)$, $X(-1,2)$, $Y(-3,0)$



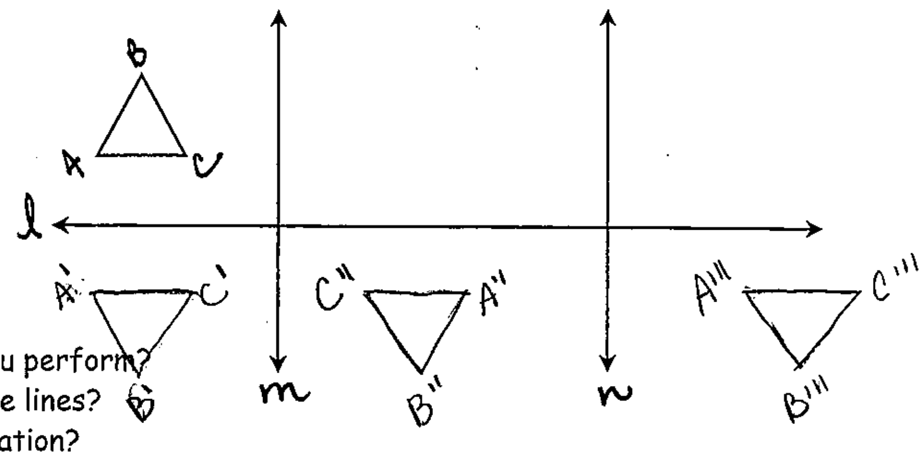
W' $(2,4)$ X' $(2,1)$ Y' $(0,3)$

12-4 Composition of Transformations (Book 12.4)

Objective: Understanding glide reflections and a composite of transformations.

Class Example:

3rd 2nd 1st
Draw $r_n(r_m(r_l(\Delta ABC)) = \Delta XYZ$



Questions:

3 — (1) How many reflections did you perform?

intersecting & parallel lines — (2) What do you notice about the lines?

(3) What happens to the orientation?

orientation switched

Here's the real definition:

A glide reflection is a composition of two isometries.

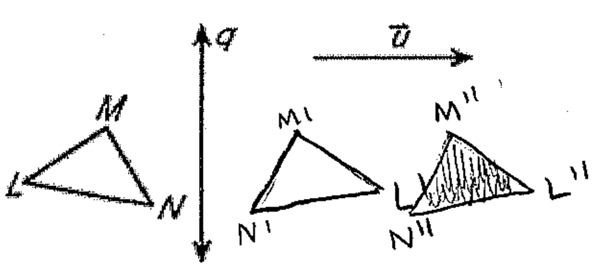
In other words: a glide reflection is a composition of a translation & reflection.

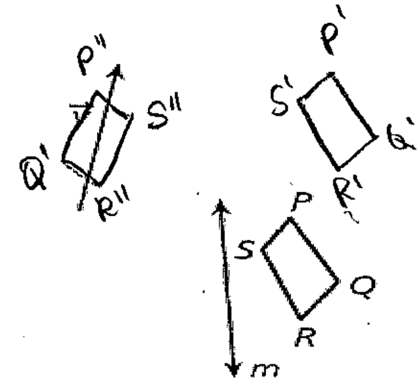
Because a glide is a composition of two isometries, then vectors can be used in place of the parallel lines.

Example: In #1-2, complete the glide reflection.

2nd 1st
1. $T_{\vec{v}} \circ r_q(LMN) = L'M'N'$

2. $r_m(T_{\vec{v}}(SPQR)) = \cancel{ABCD}$ S''P''Q''R''

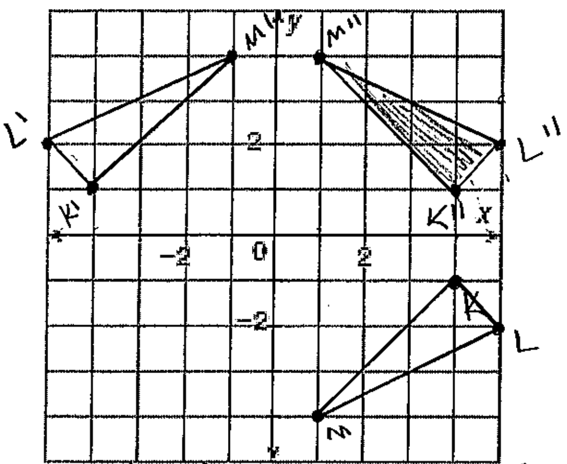




Example : In #3-4, complete the composite of transformations.

3) $r_{y\,axis} \circ R_{180} \triangle KLM$ $(-x,y)$ $(-x,-y)$

K(4,-1), L(5,-2) and M(1,-4).

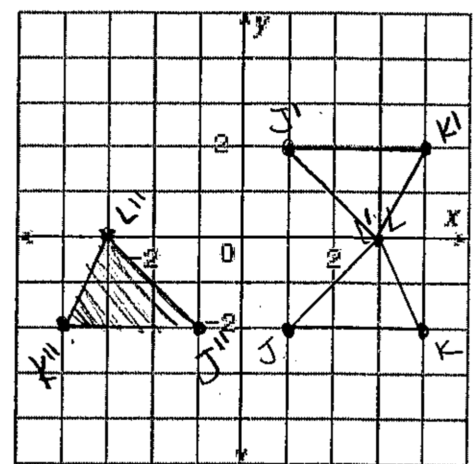


K' $(-4,1)$ L' $(-5,2)$ M' $(-1,4)$

K" $(4,1)$ L" $(5,2)$ M" $(1,4)$

4) $R_{180}\,(r_{x\,axis})\triangle JKL$ $(-x,-y)$ $(x,-y)$

J(1,-2), K(4,-2) and L(3,0).



J' $(1,2)$ K' $(4,2)$ L' $(3,0)$

J" $(-1,-2)$ K" $(-4,-2)$ L" $(-3,0)$

A translation is two reflections over parallel lines.

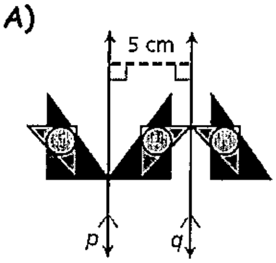
A rotation is two reflections over intersecting lines.

A glide reflection is produced by a translation and then a reflection.

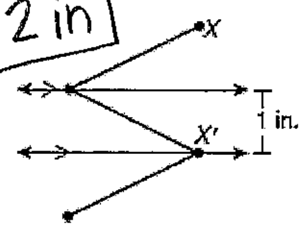
twice the distance b/t the // lines

Example #5 Determine the magnitude of each translation.

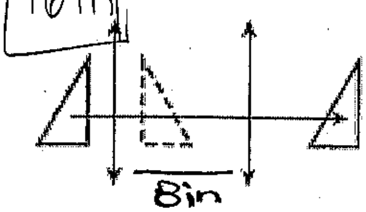
A) 10 cm



B) 2 in

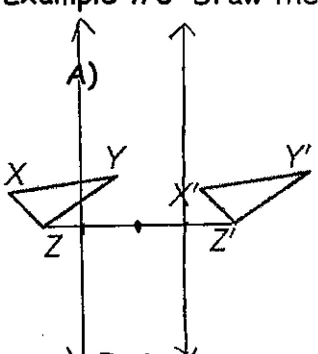


C) 16 in

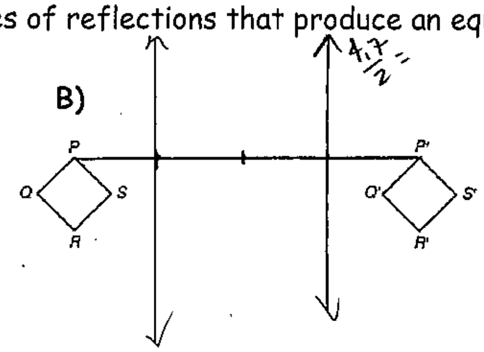


Example #6 Draw the two lines of reflections that produce an equivalent transformation.

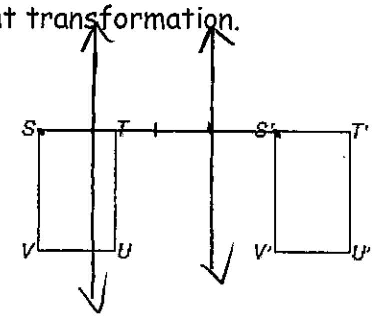
A)



B)



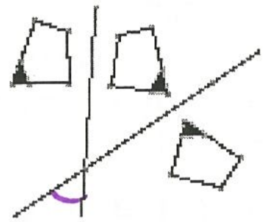
C)



① Connect a pre-image & image pt ② Bisect that line ③ ⊥ bisector of each segment

Example #7 Determine the magnitude of each rotation.

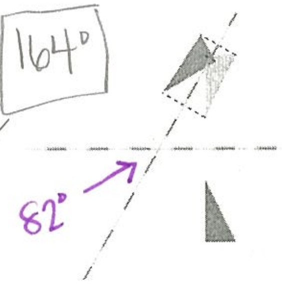
A)



$2.67 =$ $\boxed{134°}$

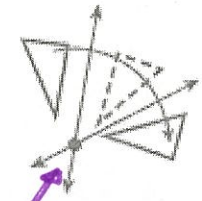
$67°$

B) $\boxed{164°}$



$82°$

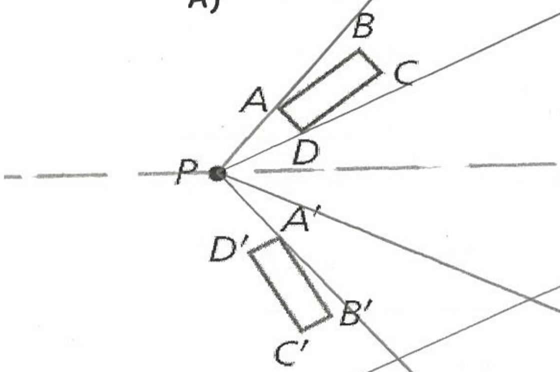
C) $88°$



$44°$

Example #8 Draw the two intersecting lines that produce the following rotation.

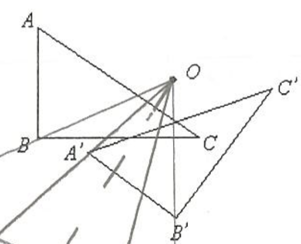
A)



$\frac{95}{2} = 47.5$

$\frac{47.5}{2} = 23.75$

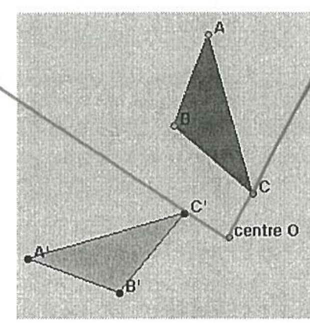
B)



$65/2 = 32.5$

$32.5/2 = 16.25$

C)



① connect the center of rotation to 2 corresponding pts

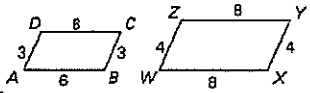
② Bisect the angle that is formed from step 1

③ Bisect each of the 2 angles created from Step 2

$\sim$ *write in corresponding order*

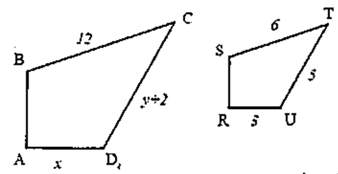
Example #6 Write a similarity statement based on each dilation.

A)



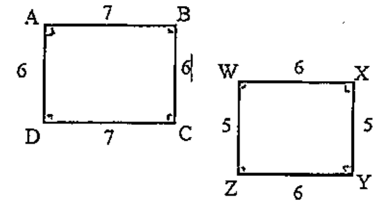
$\square ABCD \sim \square WXYZ$

B)



Quad $ABCD \sim$ Quad $RSTU$

C)



Rect $ABCD \sim$ Rect $WXYZ$

Examples: In #7-10, complete the given application question.

7. A photograph has the dimensions of 30 cm by 35 cm. If we enlarge the photograph by a scale factor of 3, what are the new dimensions?

$30 \times 3 = 90$

$35 \times 3 = 105$

$\boxed{90\text{cm by }105\text{cm}}$

8. A picture was 5" by 7" and Sue reduced it to 3.5" by 4.9". What is the scale factor that she used?

$\frac{3.5}{5} = .7$

$\frac{4.9}{7} = .7$

$\boxed{SF = .7}$

9. On a sketch of a flower, 4" represents 1" on the actual flower. If the flower has a 3" diameter in the sketch, what is the diameter of the actual flower?

$$\frac{\text{model}}{\text{actual}} \quad \frac{4''}{1''} = \frac{3''}{X}$$

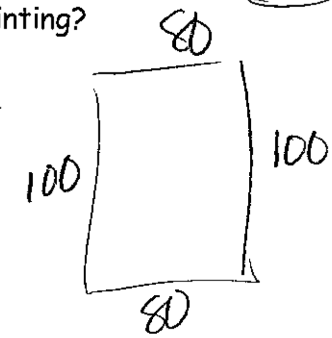
$4x = 3$

$\boxed{x = 3/4 \text{ in}}$

10. An artist is creating a large painting from a photograph by dividing the photograph into square and dilating each square by a scale factor of 4. If the photograph is 20 cm by 25 cm, what is the perimeter of the painting?

$20 \times 4 = 80$

$25 \times 4 = 100$



$= \boxed{360\text{ cm}}$

Reflections

x-axis: $(x,y) \to (x,-y)$
y-axis: $(x,y) \to (-x,y)$
$y=x$: $(x,y) \to (y,x)$

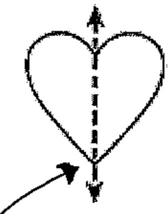
Rotations

Clockwise: negative
counterclockwise: positive
$90° / -270°$: $(x,y) \to (-y,x)$
$180°$: $(x,y) \to (-x,-y)$
$-90° / 270$: $(x,y) \to (y,-x)$

12-5 Symmetry (Book 12.5)

Objective: Identify and describe the two types of symmetry in figures.

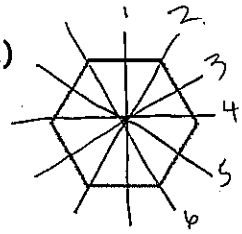
Symmetry - transformation when the image & the pre-image coincide.
(i.e. when you split something into 2 identically equal parts)



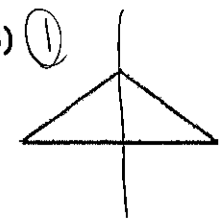
Line of Symmetry - line where the figure is divided/folded. (sometimes called the line of reflection)

Example #1 Draw all lines of symmetry for the following figures.

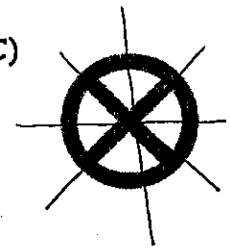
A) 6



B) 1

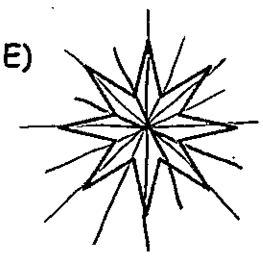


C) 4

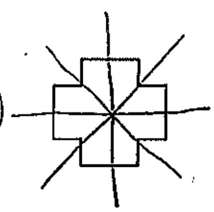


D) none ∅

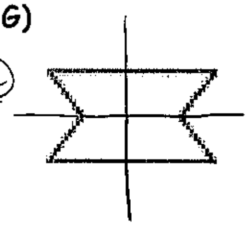
E) 8



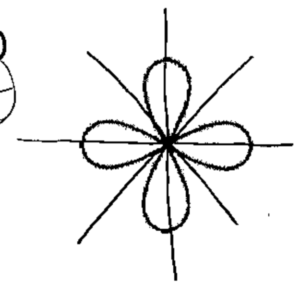
F) 4



G) 2



H) 4



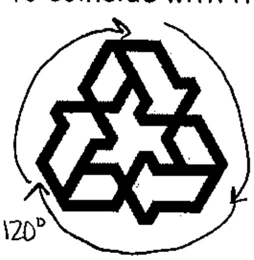
Rotational Symmetry - when a figure is rotated about a point between 0° & 360° so that the image coincides with the pre-image.

Angle of Rotational Symmetry - smallest angle which a figure can be rotated to coincide with itself.

Order - number of times the figure coincides with itself as it's rotated.

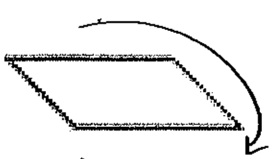
* Note -- the angle & order are very closely related.

order=3
angle $= \frac{360}{3} = 120°$



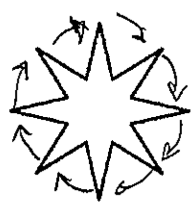
Example #2 Identify if the following figures have rotational symmetry. If they do, give the order and degree of rotational symmetry.

A)



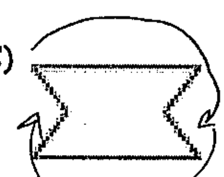
order=2
angle=180°

B)



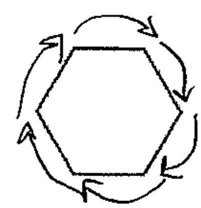
order=8
angle $= \frac{360}{8} = 45°$

C)



order=2
angle=180

D)



order=6
angle=60° 13

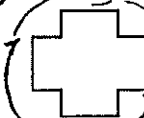
order = 3 angle = 120°

E)

none

F)

G)



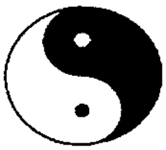
order = 4 angle = 90°

H)

none

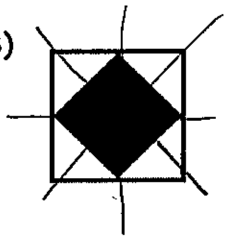
Example #3 Describe the symmetry of each figure. Draw all lines of symmetry. If there is rotational symmetry, give the angle and the order.

A)



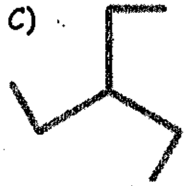
no lines
no rotational

B)



line: 4 lines
rotational: order 4
angle 90°

C)



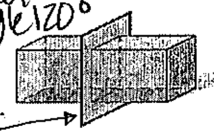
Rotational
order = 3
angle 120°

D)

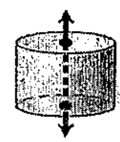
no lines
Rotational
order = 2
angle = 180°

3D Symmetry:

<u>Plane Symmetry</u> - is when a plane divides a figure in 2 halves.

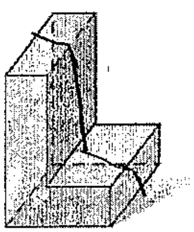


<u>Symmetry about an axis</u> - line where a figure can be rotated (or spinned).



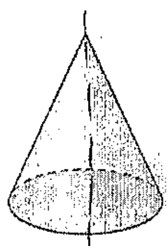
Example #4 Tell whether each figure has plane symmetry, symmetry about an axis, neither, or both.

A)



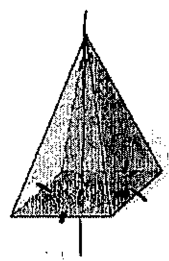
plane

B)



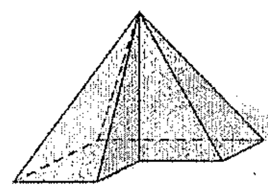
Both

C)



both

D)



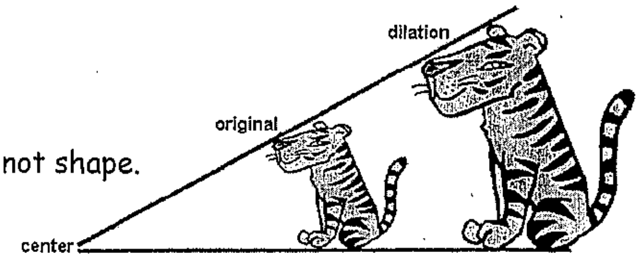
neither

12-6 Dilations (Book 12.7)

Objective: Identify and draw dilations.

Vocabulary

Dilation - a transformation that changes size, but not shape.

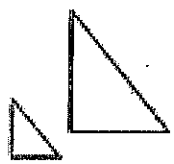


Example #1 Identifying dilations- Tell whether each transformation appears to be a dilation.

A)

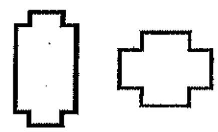
no

B)



yes

C)



no

D)

yes

→ ratio of corresponding sides

Scale Factor - how much bigger or smaller a figure becomes (k).

Enlargement (expansion) - figure that becomes bigger, k > 1.

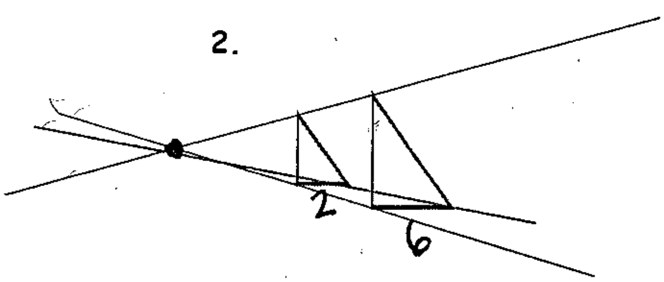
Reduction (contraction) - figure that becomes smaller, 0 < k < 1.

Identity - when a figure doesn't change size, k = 1.

Center of Dilation - the point where the dilation occurs or where the preimage & image line up.

Examples: In #2 & 3, use the dilation below to find, (a) center of the dilation, and (b) scale factor.

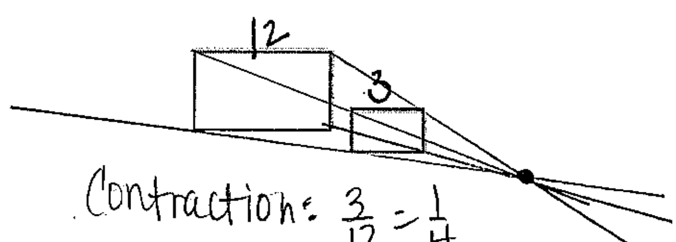
2.



expansion → $\frac{6}{2}$ or $\frac{2}{6}$ ← contraction

3 or $\frac{1}{3}$

3.

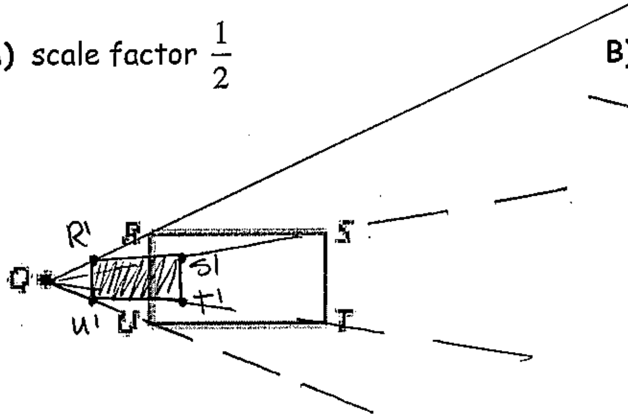


Contraction: $\frac{3}{12} = \frac{1}{4}$

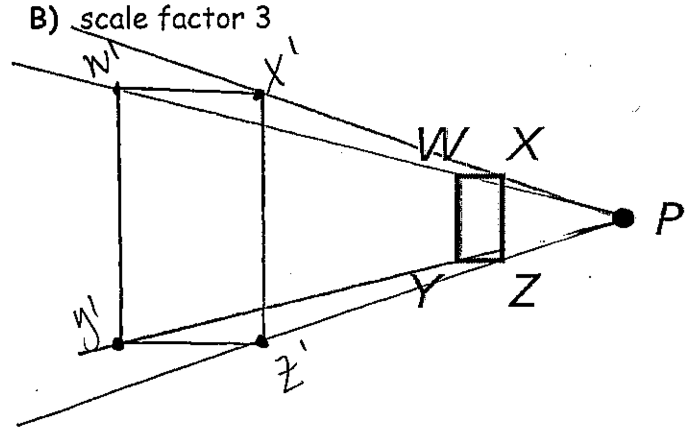
expansion: $\frac{12}{3} = 4$

Example #4 Draw the image under a dilation with the following scale factor.

A) scale factor $\frac{1}{2}$

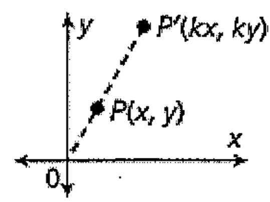


B) scale factor 3



Dilations in the Coordinate Plane

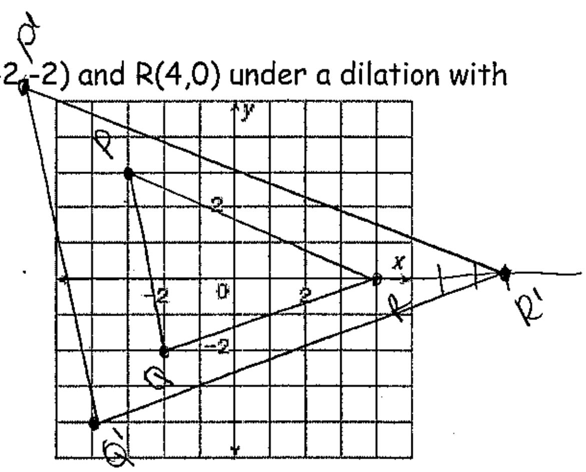
If $P(x, y)$ is the preimage of a point under a dilation centered at the origin with scale factor k, then the image of the point is $P'(kx, ky)$.



$(x, y) \rightarrow (kx, ky)$

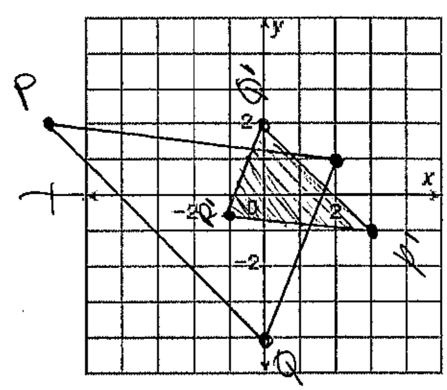
Example #5 Drawing Dilations in the Coordinate Plane

A) Draw the image of the triangle with vertices P(-3,3), Q(-2,-2) and R(4,0) under a dilation with a scale factor of 2 centered at the origin.



P' $(-6,6)$ Q' $(-4,-4)$ R' $(8,0)$

B) $D_{\frac{-1}{2}}(\Delta PQR)$ about the origin. P(-6,2), Q(0,-4), R(2,1)



P' $(3,-1)$ Q' $(0,2)$ R' $(-1,-.5)$